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INDUSTRY'S RESPONSIBILITY IN THE SHORTAGE OF ENGINEERS *

BY

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The shortage of engineers is not simply the concern of industry and educators, because it is of fundamental importance to the fabric of the American economy. The subject presents so many problems of such complex and diversified nature that any attempt on my part to analyze all the reasons for the present crisis, or suggest an absolute solution would be, to say the least, presumptuous.

Nevertheless, no one can question the self-evident fact that because of this shortage of engineers we are confronted by a most critical situation in the economic well being of our nation. No doubt this has been occasioned in large measure by the rapid expansion of our industrial activities without corresponding attention to the resulting increased demands for trained engineers. The disturbing point is that there seems no practical way of improving matters before they become worse.

Two major segments of our society have a vital interest in this situation, our educators and our industrial leaders. The educators have been especially cognizant of this dilemma during the past decade and their studies and recommendations deserve great credit. Industry, on the other hand, has only lately shown any serious concern over this situation. Even so, this concern has largely been devoted to the wishful-thinking school of reason, which concludes that the problem will somehow correct itself. We can count on the fingers of one hand those few outstanding industrialists who have demonstrated a clear-cut appreciation of the problem.

The general public has little awareness of the problem or its significance. People traditionally accept the contributions of science and

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engineering as an everyday matter. And what is more to the point, the public has come to expect an ever-increasing improvement in its standards of living. But, just as sure as we are here today, this improvement will cease to be an accomplished fact in American life if we fail to recognize the interdependence of industry and engineers to economic progress. The discouraging part of the situation is the fact that there is little that can be done now to prevent a further shortage of engineers for at least the next several years. Personally, I am not so sure that we will not continue to face a serious shortage for the next decade. Even this presupposes that definite steps must be taken throughout the nation to stem the decline in the number of engineers available to industry.

Highly trained manpower is an absolute essential to the business life of our nation, and it must keep pace with our expanding industrial economy. A backward glance will show that we have concentrated our attention on the facilities of production, and left the supply of engineering and scientific brains to our educators. We have been tremendously lucky thus far. In fact, I believe that most of our educators felt like a voice crying in the wilderness in their efforts to sound an alarm to industry of an impending critical shortage of engineers.

The urgency of the situation is compellingly dramatized by the fact that the placement office of just one school—Drexel Institute of Technology—had 8344 positions made available to graduates at a time when the school was graduating a grand total of 414 engineering and business administration students.

In order that we may have a reasonably clear appreciation of the problem, we must have an understanding of the root causes of the present emergency. Many reasons can be offered, particularly with respect to a sharp decline in the enrollment of engineering students. Back in the '30's there was much loose talk about the overcrowding of the engineering profession. Some of that talk came from industrialists themselves. These men not only underestimated the number of men wanted for strictly professional employment in engineering posts, but also disregarded or discounted industry's need for technically trained men in management, in sales, and in fact at every point where scientific knowledge is transforming our methods of production and control.

We all know that the wide publicity given these unsound forecasts in government publications, popular magazines and newspapers discouraged young people from scientific careers. And yet today, a well publicized statement estimates that while industrial research and development has increased nearly 500 per cent, the number of scientists and engineers has barely doubled.

The door has been opened to a period of industrial and commercial development unequalled in history. How short-sighted the management of American industry will be if industrial expansion is attempted

without a parallel increase of facilities for higher technological education.

For the purposes of emphasis, I quote from an authority—the Bureau of Labor Statistics—which shows that the number of professional engineers in this country has increased tenfold since 1900.

The supporting figures are interesting: 40,000 in 1900; 130,000 in 1920; 260,000 in 1940; and 400,000 in 1950.

Correspondingly, the ratio of engineers to the total number of industrial works has risen steadily from one engineer to every 255 workers in 1900 to one engineer to every 65 workers in 1950. Dean Hollister of Cornell has estimated that there was one engineer for every 50 workers last year.

The average layman might consider these figures encouraging, and indicative of a closer relationship between the man in the shop and the engineer. However, from a professional standpoint, even a casual glance at such data presents a rather alarming picture.

I am convinced that the initial step in relieving the shortage of engineers must be toward creating an atmosphere of better public understanding, and an appreciation by young people of the great opportunities in the field of engineering and scientific pursuits. An interesting illustration of this conclusion occurred about a year ago when the Drexel Institute of Technology—which has an enrollment in its engineering courses of approximately 1500 students—made a survey of Philadelphia and suburban high schools to ascertain what percentage of graduates planned to study engineering. The results showed that the percentage of graduates preparing for college had not changed during the previous year, but that there was an increase of 20 per cent in the number planning to enter engineering colleges.

The national engineering societies (A.I.E.E., A.S.M.E., N.A.M., the American Society of Engineering Education, the Engineers Joint Council, and many more) have committees on guidance and on manpower which advertise in the secondary schools not only the public need for engineers, but also the opportunities for technical and general education which are offered by colleges of engineering. These efforts have been well timed, and I'm certain will be productive of great good.

It may be that we shall have to cast about for still other ways in which to stimulate an interest among young people in the type of education that will fit them to take their places in industry. One way of encouraging young people to turn their thoughts in the direction of a technical education is the technological museum, which has not been as fully exploited as it could be. The Franklin Institute Museum, for example, is actively engaged in promoting an interest in scientific employment and industrial crafts by its special exhibitions, held periodically, for the display of work executed by students from the surrounding area. The museum's staff of demonstrators receives recruits from those who have been inspired to follow a technical education

by their visits to the museum or through these students' exhibitions. These young people find a profitable occupation in congenial surroundings while following their advanced technical training at college. Among the recent prize-winners at the exhibitions, several have taken up the study of medicine, some have become electrical engineers, and one a chemist. Since industry has become a partner in scientific education it may well be a sound idea to devote more attention to the potentialities of these technological museums where the plastic minds of youths can be molded into the forms required in our intensified industrial age.

However, this is but one phase of the problem. In addition to attracting increasing numbers of young people to the engineering profession, the colleges must be prepared to satisfactorily cope with a vastly increased student body, by way of additional instructors and expanded facilities.

If we truly wish to attract a full and adequate share of the best young brains to the studies of industrial science and industrial economics—including labor economics—then the conditions of study must be attractive. If we want qualified young men to come under the influence of sound, alert and inspiring teachers—then we dare not fail to develop our centers of higher technical education, and to make their resources and facilities comparable in all respects to the best offered by our senior universities.

Here I believe is one of the most important phases of the problem, and its solution requires the combined attention of educational administrators and industrial leaders. I do not flatter myself as one qualified to speak authoritatively on the broadening subject of engineering education. However, as a consequence of close contact with industry as well as a long standing active interest in the expansion of Philadelphia's industrial university, Drexel Institute of Technology, I am at least sure of the present shortage of engineers, and equally certain that industry—the principal cause of this shortage—does not understand the problem and thus far has evidenced a most indifferent attitude toward ways and means of aiding in its solution.

The problems of our economy cannot be solved on a national scale unless we can solve them in their smaller, local dimensions. Philadelphia, Pittsburgh, Chicago, and Detroit have each their own institutions of applied science and engineering. In each community industry has a real stake in the healthy development of schools of technology if the community is to prosper, and if the national interest is to be guarded.

I would like to make passing mention of one educational plan which has shown most favorable results, both from an educational as well as industrial viewpoint, and which represents a useful kind of partnership between education and industry. I refer, of course, to the so-called Co-operative Program, now part of the curriculum in quite a few engineering colleges. Although the course is usually five full years, the

student has the great advantage of at least having the "rough edges" eliminated by his intimate contact with industry. Of commensurate importance is the fact that on graduation it permits the student to embark on his career as a trained employee—several years ahead of where he would be without his contact with business.

There is very little in what I have voiced that is not already common knowledge to most in this audience. Nevertheless, it serves to emphasize a situation that seems quite clear to educators, quite hazy to industrialists, and is neither understood nor appreciated by the general public.

My own conclusions come from observation and experience in efforts to obtain capital funds to meet the urgent needs of an engineering college in the Philadelphia area, through contributions from local industry. We have approached the problem largely on the basis of asking for contributions toward the necessary expansion of our facilities so that the ever increasing demands made by industry itself can be supplied. I confess that progress has been slow because so many fail to understand the direct or indirect relationship of educational service to the particular organizations involved. In most cases, boards of directors and executives as well view such contributions as charitable gifts, the limit of which in many instances has already been determined by law or stockholder action.

As I view the situation, the solution requires a realistic approach in which stockholders, directors, and executives realize that their business future demands an ever increasing number of trained scientists and engineers. They should also be impressed with the fact that such personnel cannot be picked off the sidewalks. It takes years of training and costly facilities to produce a bona fide engineer.

Unfortunately, today industry's apparent well-being is taken for granted. That is one of the reasons for the mess we are now in. Whether or not you agree, we face the proposition that industry must pay for its additional scientific and engineering brains just as it now pays for its facilities of doing business. Industrial leaders must get into their heads that contributions in support of engineering and scientific education should be a direct charge against the business—not a charitable gift. Management has the responsibility of selling this to their directors and stockholders. Once this is done, a direct percentage of gross—say 1/20 of one per cent, or less—could go far toward correcting the present untenable situation. The outstanding fact is that the problem will not be solved unless and until those responsible for the prosperity of industry take the lead in its solution.

So that I will not be misunderstood, permit me again to emphasize the fact that our industrial economy is in the initial stage of an emergency. There is a definite scarcity of scientists and engineers and this scarcity will become worse rather than better during the next five years. It may well be ten years before conditions are normalized. This

situation demands action now—not five or ten years hence. Our engineering colleges are the only available sources of supply. We know there is little possibility of their increasing their student bodies without greatly expanded facilities and teaching forces, and this requires a large cash outlay. The one segment of our economy that has the greatest stake in this program is industry, and, in my opinion, industry must assume the responsibility of providing the requisite financial aid.

I repeat, we are in an emergency—but one that can be short-lived once we face it squarely. This is not a problem to be confused with the complexities of higher education generally. In simplest terms, it is a problem affecting the supply of scientific and engineering students alone—a problem which rests on the shoulders of industry, and which industry in its spectacular developments has itself created.

Electric utilities should be particularly energetic in the support of such a program. Here we have a business that depends on the industrial growth of the community for its own expansion. It not only has an individual interest in obtaining engineering personnel, but must have a vital interest in the health of business generally. I suggest that utility executives have an especial obligation for leadership in seeing to it that the needs of engineering colleges in their particular communities are met.

Let us ever be mindful of the fact that the status quo of our economy is predicated upon our ability to be equal to the challenge, as well as the opportunity. I, for one, am confident that American industry properly alerted to the problems confronting us will not fail—itself or the public.

Industry and educational institutions share in the responsibility of providing a workable solution.

The hopes of our industrial future are based upon a steady improvement in our rate of productivity and the consequent lowering of production costs and distribution. It behooves industry now and in the years ahead to take a far greater interest in education than it has in the past. Should this not occur, then industry will have failed, and the one alternative remaining is complete government subsidy and all of the evils which it portends.

Heaven protect us from such an eventuality.

AMPLITUDE VARIATIONS IN A FREQUENCY-MODULATED OSCILLATOR

BY

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ABSTRACT

Frequency modulation of a vacuum-tube oscillator often is accomplished by causing the capacitance in the circuit to vary in accordance with a modulating signal. Because of the way in which the amplitude of the alternating voltage generated by the oscillator is controlled, variation of the capacitance introduces simultaneous amplitude modulation of this voltage. The nature and amount of amplitude modulation depend upon the interrelation of several parameters of the oscillating system. An analysis is presented for this type of modulation. Limitations inherent in the analysis include maximum values for the magnitude of the change in capacitance and for the time rate of this change. Elements of the oscillator are assumed to be chosen so that the generated voltage is nearly sinusoidal. These assumptions usually apply in practice.

I. NEGATIVE-RESISTANCE OSCILLATORS

Many vacuum-tube oscillators can be described in terms of an oscillatory circuit coupled to a negative-resistance element. Because of physical limitations, any realizable negative resistance is nonlinear, becoming positive if large voltages exist across it. The significant properties of such a negative resistance can be represented by the simple equation

$$i = -ae + be^3. \tag{1}$$

In this equation, i is the instantaneous current and e is the instantaneous voltage applied to the resistance element, and both a and b are positive constants. A parallel combination of such an element together with an inductor and a capacitor can be described by the well-known van der Pol equation,

$$\ddot{x} - \alpha(1 - x^2)\dot{x} + x = 0. \tag{2}$$

If parameter α is small compared with unity, very-nearly sinusoidal oscillations occur; if parameter α is larger, relaxation oscillations exist (1).[2]

In certain systems of frequency modulation, the value of one of the reactive elements of the oscillating circuit is made to vary with time so as to produce a corresponding variation in frequency. This reactance variation is controlled by external means and may involve either a mechanical change, as produced by a condenser microphone, or an

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

equivalent variation produced by a reactance tube (2). The resulting frequency modulation is accompanied by simultaneous modulation of the amplitude of the alternating voltage. This amplitude modulation is undesirable, and in most cases it is removed by limiting circuits following the oscillator.

In the present discussion an analysis is given for the modulation of a negative-resistance oscillator of the type just described. The nature of the analysis is such that it is limited to cases of frequency deviation small compared with the unmodulated frequency. This condition usually applies in practice. Frequency modulation produced by variation of reactance comes about in rather simple fashion. Amplitude modulation, occurring simultaneously, depends upon the interrelation of several parameters of the circuit, and is a more complicated phenomenon. The analysis shows the importance of these various parameters in their effect upon the resulting amplitude modulation.

II. CHARGE IN CIRCUIT FREE OF RESISTANCE

In setting up an analysis for the modulated oscillator, it is simplest to consider first a problem that has been studied in detail (3, 4, 5). This is the case of an oscillating system that is free of resistance, either positive or negative. An approximate analysis of this system leads directly to the modulated oscillator which is considered here. Such a circuit consists merely of an inductance L and a capacitance C_0 connected in series. It can be described by the equation

$$L\ddot{q} + q/C_0 = 0, \tag{3}$$

where q is the instantaneous charge on the capacitor. If both L and C_0 are constant quantities, the charge may vary as a sinusoidal function of time,

$$q = A \sin (\omega_0 t + \theta). \tag{4}$$

The angular frequency is constant at $\omega_0 = (LC_0)^{-\frac{1}{2}}$. The amplitude A and phase angle θ also are constants and are determined by the initial conditions.

Frequency modulation of this free oscillation can be achieved by introducing a time variation of the capacitance. For example, the instantaneous capacitance might be given by the equation

$$C = C_0[1 + \epsilon\phi(t)] \tag{5}$$

applying for positive times, $t \geq 0$. The capacitance C_0 is assumed to exist for negative times, $t < 0$. Function $\phi = \phi(t)$ describes the way in which the capacitance varies with time. For the purposes of the analysis, it is necessary to assume that the relative change in ϕ during any single cycle of the oscillation is small. This requirement can be

expressed mathematically as

$$d\phi/\phi = (\dot{\phi}/\phi)(2\pi/\omega_0) \ll 1,$$
$$\dot{\phi} \ll \phi(\omega_0/2\pi).$$

In addition, it is required that the higher-order derivatives of ϕ also are very small, and in particular

$$\ddot{\phi} \ll \dot{\phi}(\omega_0/2\pi) \ll \phi(\omega_0/2\pi)^2. \tag{6}$$

Furthermore, in order to prevent a jump in capacitance at zero time, it is necessary that at $t = 0$, $\phi(0) = 0$. Finally, parameter ϵ in Eq. 5 determines the magnitude of the change in capacitance. For the purposes of the analysis, it is necessary to assume that $\epsilon \ll 1$. These requirements upon the rate and magnitude of the relative variation of capacitance are satisfied in most vacuum-tube oscillators that are subjected to frequency modulation.

If Eqs. 3 and 5 are combined, the result can be written as

$$\ddot{q} + \omega_0^2 q = -\epsilon\phi\ddot{q}. \tag{7}$$

This equation with a time-varying coefficient may be considered as a form of Hill's equation, for which exact solutions sometimes can be found. However, if the assumed condition of $\epsilon \ll 1$ applies, an approximate solution can be obtained simply through the method of variation of parameters (6). The initial step in this process is to find the generating solution, obtained for $\epsilon = 0$. This generating solution is merely Eq. 4. The value of the instantaneous charge for this solution is designated as q_0.

If, now, ϵ becomes larger than zero, but remains small compared with unity, changes occur in both amplitude A and phase angle θ of Eq. 4. The changes in amplitude and angle are given by the relations (7)

$$\dot{A} = -(\epsilon/\omega_0)\phi\ddot{q}_0 \cos(\omega_0 t + \theta), \tag{8}$$

$$\dot{\theta} = (\epsilon/A\omega_0)\phi\ddot{q}_0 \sin(\omega_0 t + \theta). \tag{9}$$

Substitution of the value of q_0 from Eq. 4 into Eq. 8, followed by the use of the formula for the sine of the double angle, gives

$$\dot{A} = \tfrac{1}{2}\epsilon\omega_0 A\phi \sin 2(\omega_0 t + \theta). \tag{10}$$

The solution for amplitude A is

$$A = B \exp\left[\tfrac{1}{2}\epsilon\omega_0 \int \phi \sin 2(\omega_0 t + \theta)dt\right], \tag{11}$$

where B is a constant determined by initial conditions. Subject to the conditions of Eq. 6, the integral appearing in Eq. 11 is approximately $-(1/2\omega_0)\phi \cos 2(\omega_0 t + \theta)$, and thus,

$$A = B \exp [-(\epsilon/4)\phi \cos 2(\omega_0 t + \theta)]. \tag{12}$$

Furthermore, under the conditions of Eq. 6, amplitude A varies only slowly from cycle to cycle of the oscillation of charge q. Therefore, the values of A that are of primary interest are those which occur at the times of maximum charge during a cycle. These values occur very near the instants for which $\sin(\omega_0 t + \theta) = \pm 1$ in Eq. 4. At these instants, $\cos 2(\omega_0 t + \theta) = -1$ in Eq. 12. Thus, the amplitude of the charge is given by

$$A = B \exp(\epsilon\phi/4), \tag{13}$$

or if $\epsilon \ll 1$, as assumed

$$A = A_0(1 + \epsilon\phi/4), \tag{14}$$

where A_0 is the amplitude at the time $t = 0$.

In a similar fashion Eqs. 4 and 9 can be combined to give

$$\dot{\theta} = -\tfrac{1}{2}\epsilon\omega_0\phi[1 - \cos 2(\omega_0 t + \theta)]. \tag{15}$$

Here it is sufficient to consider only the average value of the quantity in the bracket during a complete cycle. Since the average of the cosine is zero, the average value of $\dot{\theta}$ is

$$\dot{\theta} = -\tfrac{1}{2}\epsilon\omega_0\phi. \tag{16}$$

If Eqs. 4, 14, and 16 are combined, the instantaneous charge is found to be given by

$$q = A_0(1 + \epsilon\phi/4) \sin\left[\omega_0 t - \tfrac{1}{2}\epsilon\omega_0 \int \phi dt + \theta_0\right]. \tag{17}$$

Amplitude A_0 and angle θ_0 depend upon initial conditions.

As a simple example, the capacitance might be assumed to vary as a sinusoidal function of time, so that $\phi = \sin \omega_1 t$, where it is necessary that $\omega_1 \ll \omega_0$. For this case, the instantaneous charge is

$$q = A_0(1 + \tfrac{1}{4}\epsilon \sin \omega_1 t) \sin [\omega_0 t + \epsilon(\omega_0/2\omega_1) \cos \omega_1 t], \tag{18}$$

and the instantaneous angular frequency is

$$\omega = \omega_0(1 - \tfrac{1}{2}\epsilon \sin \omega_1 t). \tag{19}$$

With this sine function for $\phi(t)$, and with $\epsilon \ll 1$, Eq. 7 is equivalent to a Mathieu equation. Subject to the condition, $\omega_1 \ll \omega_0$, stable solutions exist for the Mathieu equation with known form (**8**). These solutions can be reduced to the forms given in Eqs. 18 and 19.

III. VOLTAGE IN CIRCUIT FREE OF RESISTANCE

The instantaneous charge on the capacitor in the oscillating system is given by the preceding analysis culminating in Eq. 17. The instantaneous voltage, e, across the capacitor could be found as

$$e = q/C, \tag{20}$$

where instantaneous values of q and C are used. On the other hand, in the succeeding analysis of the negative-resistance oscillator, it is necessary to work with the voltage directly. For this reason, it is useful to consider the voltage for the resistanceless circuit. The voltage across the capacitance is given by the following equations, analogous to Eq. 3,

$$\begin{aligned} d(Ce)/dt + (1/L)\int e dt &= 0, \\ \dot{C}e + 2C\dot{e} + C\ddot{e} + e/L &= 0, \end{aligned} \tag{21}$$

where time variation of C is allowed. If this variation of capacitance is described by Eq. 5, as before, then Eq. 21 becomes

$$\ddot{e} + \omega_0^2 e = -\epsilon(\ddot{\phi}e + 2\dot{\phi}\dot{e} + \phi\ddot{e}), \tag{22}$$

which is analogous to Eq. 7. An approximate solution for this equation can be found as before, the first step being

$$e_0 = A \sin(\omega_0 t + \theta). \tag{23}$$

Amplitude A and angle θ are, of course, different in general from the values of these symbols appearing in Eq. 4 for the charge. If Eqs. 22 and 23 are used with an equation corresponding to Eq. 8, the result is

$$\begin{aligned} \dot{A} &= -(\epsilon A/\omega_0)[(\ddot{\phi} - \omega_0^2\phi)\sin(\omega_0 t + \theta) \\ &\qquad + 2\dot{\phi}\omega_0 \cos(\omega_0 t + \theta)]\cos(\omega_0 t + \theta) \\ &\equiv -(\epsilon A/\omega_0) f(t). \end{aligned} \tag{24}$$

In the process of solving for A, the integral of the function $f(t)$ with respect to time is required. This integral, subject to the conditions of Eq. 6, is approximately

$$\int f(t)dt = \omega_0\phi[1 + \tfrac{1}{4}\cos 2(\omega_0 t + \theta)]. \tag{25}$$

In view of Eq. 25, the solution of Eq. 24 is

$$A = B \exp(-3\epsilon\phi/4), \tag{26}$$

where $\cos 2(\omega_0 t + \theta)$ is evaluated at the instants when $\sin(\omega_0 t + \theta) = \pm 1$. If $\epsilon \ll 1$, as assumed,

$$A = A_0(1 - 3\epsilon\phi/4), \tag{27}$$

where A_0 is the amplitude at $t = 0$.

When θ is found for the voltage, in a manner analogous to that used for the charge, the same result is obtained as given in Eq. 16. Therefore

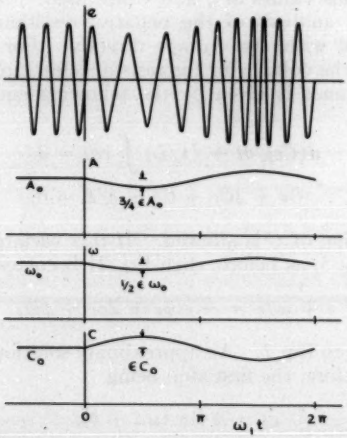

FIG. 1. Effects produced by a small sinusoidal variation of capacitance in a resistanceless oscillating circuit. Instantaneous values of capacitance C, and voltage e, as well as instantaneous amplitude A and angular frequency ω, of the voltage, are shown. The variations are somewhat exaggerated, especially in the waveform of voltage e.

the instantaneous voltage is

$$e = A_0(1 - 3\epsilon\phi/4) \sin\left[\omega_0 t - \tfrac{1}{2}\epsilon\omega_0 \int \phi dt + \theta_0\right]. \tag{28}$$

Amplitude A_0 and angle θ_0 depend upon initial conditions, and are different from the similar quantities appearing in Eq. 17.

If the capacitance is assumed to vary sinusoidally with time, so that $\phi = \sin \omega_1 t$ with $\omega_1 \ll \omega_0$, the operation of the circuit is illustrated in Fig. 1. Instantaneous values of capacitance C, voltage e, and of ampli-

tude A and angular frequency ω of the voltage, are plotted. All of the changes are exaggerated in order to show the effects that are produced. The amplitude of the charge, not plotted in the figure, varies exactly out of phase with the voltage amplitude.

IV. OSCILLATING CIRCUIT WITH RESISTANCE

The equation for the negative-resistance oscillator with a time-varying capacitance can be obtained from Eq. 22 with the introduction of the resistance characteristics of Eq. 1. The resulting equation may be written as

$$\ddot{e} + \omega_0^2 e = \alpha\omega_0(1 - \beta e^2)\dot{e} - \epsilon(\ddot{\phi}e + 2\dot{\phi}\dot{e} + \phi\ddot{e}), \tag{29}$$

where $\alpha \equiv a/C_0\omega_0$, $\beta \equiv 3b/a$, and $\omega_0 \equiv (LC_0)^{-\frac{1}{2}}$. If $\epsilon = 0$, then Eq. 29 is a form of the van der Pol equation, Eq. 2. As is the case for the van der Pol equation, a solution for Eq. 29 can be obtained analytically only if the value of parameter α is small compared with unity. This condition of $\alpha \ll 1$, however, is required if the oscillation is to be essentially sinusoidal in character. Usually it is desirable to maintain a nearly sinusoidal voltage in an actual oscillator of this kind, so that typical values of α are small. In addition to this requirement on the value of α, the value of ϵ and the rate of variation of ϕ must satisfy the conditions used in the analysis of the resistanceless circuit.

If both $\alpha \ll 1$ and $\epsilon \ll 1$, the first step in the solution of Eq. 29 is merely Eq. 23. If Eq. 23 and Eq. 29 are used in an equation corresponding to Eq. 8, the result is

$$\dot{A} = \alpha\omega_0 A[1 - \beta A^2 \sin^2(\omega_0 t + \theta)]\cos^2(\omega_0 t + \theta) - (\epsilon A/\omega_0)f(t), \tag{30}$$

where $f(t)$ is the same quantity appearing in Eq. 24. It is sufficient to take the average of the first term of Eq. 30 over a cycle so as to obtain

$$\dot{A} = \tfrac{1}{2}\alpha\omega_0 A(1 - \beta A^2/4) - (\epsilon A/\omega_0)f(t). \tag{31}$$

Because of the term in A^3, this equation is nonlinear. It can be solved by variation of parameters, for which the first step is the generating solution obtained by disregarding the nonlinear term. The generating solution is

$$A = B\exp(\tfrac{1}{2}\alpha\omega_0 t - 3\epsilon\phi/4) \equiv B\exp[\mu(t)], \tag{32}$$

where $\int f(t)dt$ is evaluated as in Eqs. 25 and 26.

In order to consider the effect of the nonlinear term in Eq. 31, coefficient B is allowed to vary so that

$$(dB/dt)\exp[\mu(t)] = -(\alpha\beta\omega_0/8)B^3\exp[3\mu(t)]. \tag{33}$$

Variables B and t can be separated, use made of the condition $\epsilon \ll 1$, and the equation integrated approximately to obtain

$$B = 2\beta^{-\frac{1}{2}}[\exp(\alpha\omega_0 t) - (3\epsilon\alpha\omega_0/2)\Phi(t) + 4k/\beta]^{-\frac{1}{2}}, \tag{34}$$

where $\Phi(t) \equiv \int \phi(t) \exp(\alpha\omega_0 t)dt$, and k is a constant of integration. When this value of B is inserted into Eq. 32, the result is

$$A = \frac{2\beta^{-\frac{1}{2}} \exp(\frac{1}{2}\alpha\omega_0 t)(1 - 3\epsilon\phi/4)}{[\exp(\alpha\omega_0 t) - (3\epsilon\alpha\omega_0/2)\Phi(t) + 4k/\beta]^{\frac{1}{2}}}, \tag{35}$$

where use has been made of the condition $\epsilon \ll 1$.

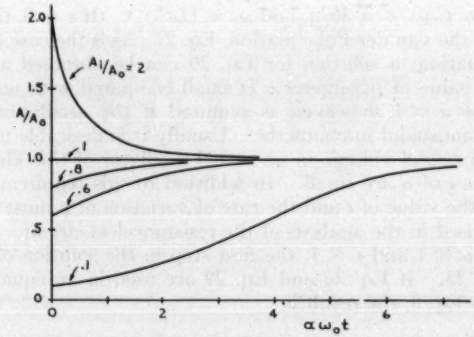

FIG. 2. Variation of the amplitude A of the voltage in a negative-resistance oscillator for several initial amplitudes, A_i. The amplitude always approaches the steady-state value $A_0 = 2\beta^{-\frac{1}{2}}$ at a rate dependent upon the product $\alpha\omega_0$.

If there is no variation of the capacitance, parameter ϵ is zero, and Eq. 29 is the van der Pol equation. The approximate solution, Eq. 35, becomes

$$A = 2\beta^{-\frac{1}{2}}[1 + (4k/\beta)\exp(-\alpha\omega_0 t)]^{-\frac{1}{2}}. \tag{36}$$

If, now, at $t = 0$ the amplitude is assumed to have the initial value, $A = A_i$, then constant k can be evaluated and used in Eq. 36 to give

$$A = A_0[1 - (1 - A_0^2/A_i^2)\exp(-\alpha\omega_0 t)]^{-\frac{1}{2}}, \tag{37}$$

where $A_0 \equiv 2\beta^{-\frac{1}{2}}$.

This Eq. 37 is the well-known solution for the change of amplitude with time in the van der Pol equation. The change in amplitude per cycle depends upon the value of parameter α. For reference purposes, several curves are plotted in Fig. 2, showing the variation of amplitude

as a function of the dimensionless time variable $\alpha\omega_0 t$. So long as the initial amplitude, A_i, is not zero, the amplitude ultimately approaches the final value, A_0.

If variation of the capacitance does occur, the conditions of Eq. 5 limit variation to positive times only. Thus, if the oscillator has been allowed to operate for a considerable interval before time $t = 0$, the amplitude will have achieved its normal steady-state value, $A_0 = 2\beta^{-\frac{1}{2}}$. This condition, $A = A_0$ at $t = 0$, can be used to evaluate constant k in Eq. 35 for the modulated oscillator. Its value is found to be

$$4k/\beta = (3\epsilon\alpha\omega_0/2)\Phi(0), \tag{38}$$

where use has been made of the requirement that at $t = 0$, $\phi(0) = 0$. The quantity $\Phi(0)$ is function $\Phi(t)$ evaluated at $t = 0$.

When the value of k from Eq. 38 and the condition $\epsilon \ll 1$ are used in Eq. 35, the result is

$$A = A_0(1 - (3\epsilon/4)\{\phi(t) - \alpha\omega_0 \exp(-\alpha\omega_0 t)[\Phi(t) - \Phi(0)]\}). \tag{39}$$

This equation gives the amplitude of the oscillating voltage as a function of the parameters of the system, for values of time greater than zero.

Data from Eqs. 23 and 29 can be used in an equation corresponding to Eq. 9 in order to find the variation in phase angle θ. Subject to the conditions imposed by the analysis, θ is found to be given by Eq. 16, once more, so that the instantaneous frequency is given by Eq. 19.

V. SINUSOIDAL VARIATION OF CAPACITANCE

If the capacitance varies sinusoidally with time, the instantaneous capacitance for positive times is

$$\left.\begin{aligned} C &= C_0(1 + \epsilon \sin \omega_1 t) \qquad \text{where} \qquad \omega_1 \ll \omega_0. \\ \phi(t) &= \sin \omega_1 t \\ \Phi(t) &= \exp(\alpha\omega_0 t)\,\frac{\alpha\omega_0 \sin \omega_1 t - \omega_1 \cos \omega_1 t}{\alpha^2\omega_0^2 + \omega_1^2} \\ \Phi(0) &= -\omega_1/(\alpha^2\omega_0^2 + \omega_1^2). \end{aligned}\right\} \tag{40}$$

When these conditions are used in Eq. 39, the resulting amplitude is found to be

$$A = A_0\{1 - (3\epsilon/4)[(1 + x^2)^{-\frac{1}{2}} \sin(\omega_1 t + \varphi) - x(1 + x^2)^{-1} \exp(-\alpha\omega_0 t)]\}, \tag{41}$$

where $x \equiv \alpha\omega_0/\omega_1$ and $\tan \varphi = x$. The instantaneous angular frequency is

$$\omega = \omega_0(1 - \tfrac{1}{2}\epsilon \sin \omega_1 t). \tag{42}$$

It is evident from Eq. 42 that the modulation of the instantaneous frequency is in accordance with the variation in capacitance, within the degree of approximation of the solution. The variation in amplitude, given by Eq. 41, depends upon parameter x. This parameter is a measure of the relative values of the rates of change introduced by the tendency of the oscillator to maintain constant amplitude, and by the capacitance variation. While upper limits must be placed upon the values of both ω_1/ω_0 and α, parameter x can have any value. If parameter x is large, only small amplitude variation occurs, while phase angle φ is large.

The variation of capacitance C, voltage amplitude A, and instantaneous angular frequency ω, are shown in Fig. 3. Only the deviations

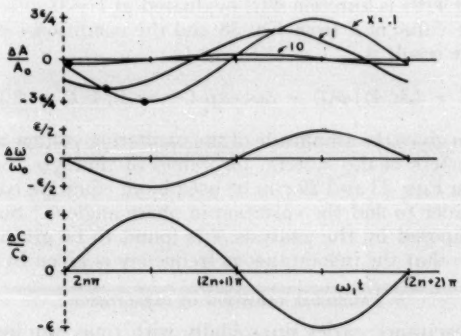

FIG. 3. Effects produced by a small sinusoidal variation of capacitance in a negative-resistance oscillator. Deviations from the mean values of the instantaneous capacitance C, voltage amplitude A, and angular frequency ω, are shown. Parameter x is defined as $x = \alpha\omega_0/\omega_1$. A steady state is assumed to exist.

about the mean values of these quantities are plotted, so as to emphasize the changes that occur. The transient represented by the last term of Eq. 41 is not considered, so that the curves represent steady-state operation. The variation of amplitude for three values of parameter x is shown in order to illustrate the effect of this parameter upon the magnitude and phase angle. Small circles represent the peaks of sinusoidal changes.

The abscissa of Fig. 3 is the dimensionless time variable $\omega_1 t$. If the modulating frequency is increased, a cycle of the sinusoid occurs more quickly and the interval plotted in the figure represents a shorter time. If product $\alpha\omega_0$ remains constant, parameter x is then decreased and there is a larger variation in amplitude occurring more nearly in exact phase opposition to the variation in capacitance.

VI. EXPONENTIAL VARIATION OF CAPACITANCE

The analysis does not apply if the capacitance changes discontinuously in time, as would be the case for a step type of variation. However, the capacitance may be assumed to vary as an exponential function of time so long as the rate of change is not too great. For such an exponential change, the instantaneous capacitance for positive times is

$$\left.\begin{aligned} C &= C_0\{1 + \epsilon[1 - \exp(-\lambda t)]\} \qquad \text{where} \qquad \lambda \ll \omega_0 \\ \phi(t) &= 1 - \exp(-\lambda t) \\ \Phi(t) &= (\alpha\omega_0)^{-1} \exp(\alpha\omega_0 t) - (\alpha\omega_0 - \lambda)^{-1} \exp(\alpha\omega_0 t - \lambda t) \\ \Phi(0) &= (\alpha\omega_0)^{-1} - (\alpha\omega_0 - \lambda)^{-1}. \end{aligned}\right\} \tag{43}$$

When these equations are used in Eq. 39, the resulting amplitude is found to be

$$A = A_0\{1 - (3\epsilon/4)(y - 1)^{-1}[\exp(-\lambda t) - \exp(-\alpha\omega_0 t)]\}, \tag{44}$$

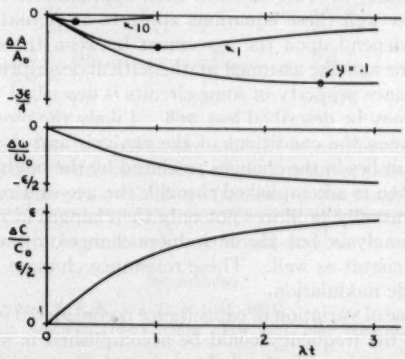

FIG. 4. Effects produced by a small exponential variation of capacitance in a negative-resistance oscillator. Deviations from the initial values of the instantaneous capacitance C, voltage amplitude A, and angular frequency ω, are shown. Parameter y is defined as $y \equiv \alpha\omega_0/\lambda$. The deviation in voltage amplitude ultimately returns to zero.

where $y \equiv \alpha\omega_0/\lambda$. The instantaneous angular frequency is

$$\omega = \omega_0\{1 - \tfrac{1}{2}\epsilon[1 - \exp(-\lambda t)]\}. \tag{45}$$

For this exponential case, parameter y determines the amplitude modulation. It is a quantity quite similar to parameter x of the sinusoidal case in that if y is large, only small changes in amplitude occur.

The variation of capacitance C, voltage amplitude A, and instantaneous angular frequency ω, are shown in Fig. 4. Again only devia-

tions from the initial value of these quantities are plotted, and the abscissa is the dimensionless time variable λt, analogous to the variable $\omega_1 t$, used in Fig. 3. The variation in amplitude for three values of parameter y is shown. If y is small, a large change in amplitude takes place and the amplitude reassumes its steady-state value relatively slowly. For example, the interval plotted in the figure is too short to show the readjustment of the amplitude for the case of $y = 0.1$. Small circles in the figure indicate the maximum deviation of the amplitude. If exponent λ is increased, the modulating exponential changes more rapidly so that the interval plotted in the figure represents a shorter time. If the product $\alpha\omega_0$ is constant, parameter y is then decreased and there is a larger variation in amplitude.

VII. APPLICATION TO ACTUAL OSCILLATOR

As was stated at the beginning of the discussion, it is necessary to assume a relatively simple mathematical description of the negative-resistance oscillator in order to allow even approximate solutions to be obtained. How well these equations apply to an actual vacuum-tube oscillator will depend upon the agreement between the characteristics of that oscillator and the assumed mathematical description of it. The negative-resistance property of some circuits is described well by Eq. 1; other circuits may be described less well. Likely the most pronounced difference between the conditions of the analysis and those existing in the actual circuit lies in the changes produced by the modulating device. Modulation often is accomplished through the use of a reactance tube. Such a circuit usually produces not only the changes in reactance considered in the analysis, but also introduces changes in the resistance of the oscillating circuit as well. These resistance changes produce additional amplitude modulation.

Only the case of variation of capacitance is considered in the analysis. Modulation of the frequency could be accomplished in similar fashion through the variation of the inductance of the oscillating circuit. Qualitative effects produced by these two methods of modulation should be similar. Because of the different way in which capacitance and inductance enter the equation for the oscillator, likely there would be quantitative differences in the solutions for the two cases.

An example illustrating numerical values of some of the quantities of the preceding analysis follows. An experimental negative-resistance element, consisting of a vacuum-tube feedback circuit, has been measured in the laboratory and found to be described by Eq. 1 with coefficients $a = 2 \times 10^{-4}$ mho (equivalent to a negative resistance of 5000 ohms) and $b = 4 \times 10^{-6}$ amp/volt^3. When this resistance element is used in an oscillator, the steady-state amplitude is $A_0 = 2(a/3b)^{\frac{1}{2}} = 8.2$ volts.

In a typical application, the angular frequency of oscillation is $\omega_0 = 10^7$ rad/sec, or $f_0 = \omega_0/2\pi = 1.6 \times 10^6$ cps. If the wave form of the oscillations is to be essentially sinusoidal, a reasonable maximum value for parameter α is in the order of $\alpha = 0.1$, or the mean capacitance of the oscillating circuit is $C_0 = a/\alpha\omega_0 = 200\ \mu\mu$f. A typical value for the maximum deviation of frequency from the mean value is $\Delta f = 80 \times 10^3$ cps., so that $\Delta f/f_0 = 0.05$, and factor ϵ is $\epsilon = 2(\Delta f/f_0) = 0.1$. The necessary deviation in capacitance is $\Delta C = \epsilon C_0 = 20\ \mu\mu$f.

The resulting variation in amplitude depends upon the nature of the modulating signal. The simplest possibility is that of sinusoidal modulation, in which case the amount of amplitude variation depends upon the angular frequency, ω_1, of the modulating signal. Parameter x is $x = \alpha\omega_0/\omega_1$ and the relative change in amplitude is $\Delta A/A_0 = (3\epsilon/4)(1 + x^2)^{-\frac{1}{2}}$, with the phase angle given by $\tan \varphi = x$. Numerical values for several modulating frequencies appear in the following table.

ω_1 rad/sec	f_1 cps.	x	$\Delta A/A_0$, per cent	φ, degrees
10^6	1.6×10^5	1	5.3	45
10^5	1.6×10^4	10	0.75	6
10^4	1.6×10^3	100	0.075	0.6

It is evident from the table that the amount of amplitude variation is relatively small, for the chosen value of parameter α, unless the modulating frequency is a reasonably large fraction of the mean frequency of oscillation. If parameter α were smaller or if the deviation Δf were larger, a larger variation in amplitude would result.

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Columbia University to Translate Russian Science Reports.—The National Science Foundation has entered into a contract with the Slavic Language Department of Columbia University, New York, to translate approximately 1000 pages of current Russian research reports in physics during the coming year. The Columbia project is part of a larger program undertaken by the Foundation to make it easier for American scientists to keep informed of current scientific developments abroad.

In the course of preparing translations, the staff will also compile files of new or unusual Russian terms in physics and related sciences as the basis for an improved Russian-English scientific glossary. During the past year the Columbia group undertook preparation of a similar glossary of terms used in metallurgy.

Limited numbers of the completed translations will be printed at the facilities maintained at Oak Ridge by the U. S. Atomic Energy Commission. In addition to the official distribution to Federal agencies interested in the material, copies will be distributed to some 40 depository libraries throughout the country which hold complete files of AEC unclassified reports. Copies may also be purchased for a nominal charge at the Office of Technical Services, Department of Commerce, Washington 25, D. C. Titles will be announced in the *Bibliography of Technical Reports*, issued monthly by OTS, and in the *Monthly List of Russian Accessions*, issued by the Library of Congress. The translations concerning nuclear physics will also be announced in *Nuclear Science Abstracts*.

The first group of translations will include physics reports from the January 1953 issues of *Doklady Akademii Nauk SSSR* (Reports of the Academy of Science of the USSR).

Rotary Switch Uses Printed Circuit Techniques.—The space- and weight-saving advantages of low-cost printed circuit techniques have been applied to a new 60-position rotary switch recently announced by the Shallcross Manufacturing Company, Collingdale, Pa. Stator contacts of the new switch are silver- and rhodium-plated copper printed on a paper-base phenolic deck only 3 in. square. Connections are made to stator contacts through easily soldered eyelet terminals. The combination of printed contacts and eyelet terminals makes this switch extremely light in weight considering its great number of contacts.

The new Shallcross switch features an isolated shaft and is designed for 4-point spacer mounting so that a number of these single pole units may be readily ganged. Contacts are arranged to make-before-break.

The Safety Belt Vindicated.—Not long ago, astonished airline stewardesses were confronted with mutiny. Many passengers refused to fasten safety belts because they insisted they were safer without them. Why? They'd "read an article" in a popular American scientific publication that said so. It all stemmed from the report of a British M.D. who had studied injuries of 28 victims of a London airline crash; stated that more than half were killed by jackknifing across their safety belts and bursting internal organs. Cornell University, contracted by the Armed Services to investigate, has completely vindicated "defendant" safety belt and restated its worth. Items: (1) bolts in the British aircraft were too weak to have caused injury before giving way; (2) in German tests where men were thrown against safety belts much more forcibly, only one suffered ill effect—a slight case of nausea that was gone within an hour.

TEM WAVES IN CYLINDRICAL SYSTEMS

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

ABSTRACT

A general theory is developed for TEM waves guided by any form of cylindrical conductors. The vector Laplacian in general curvilinear coordinates leads to very simple equations for **E** and **H**.

With electromagnetic waves, a scalar potential does not exist, since curl $\mathbf{E} \neq 0$. But if $\mathbf{E} \cdot \text{curl}\, \mathbf{E} = 0$, a scalar quasipotential Φ can be introduced, from which the **E**-vector can be obtained. The use of a quasipotential constitutes a second method for handling TEM waves.

The paper produces no new practical results; but it does develop general equations that are free from the ambiguities existing in previous treatments.

INTRODUCTION

Transverse electromagnetic (TEM) waves occur on transmission lines and with certain waveguides. Though the practical applications are understood by most radio engineers, the basic theory seems to have remained in a rather illogical and unsatisfactory state. The purpose of this paper is to present a rigorous theory for TEM waves guided by any form of cylindrical conductors. A new equation is developed which gives simple solutions for even the most complicated coordinate systems.

GENERAL SOLUTION

Consider an orthogonal cylindrical coordinate system (u^1, u^2, u^3), with u^3 (or z) parallel to the generators of the cylinders. Metric coefficients are g_{11}, g_{22}, g_{33}. It is well known (1) [3] that g_{11} and g_{22} for any cylindrical coordinate system are independent of z, and that g_{33} is always unity:

$$g_{11} = g_{11}(u^1, u^2), \qquad g_{22} = g_{22}(u^1, u^2), \qquad g_{33} = 1. \tag{1}$$

Consider a waveguide consisting of two metal cylinders, $u^1 = a$ and $u^1 = b$, insulated from each other and conforming to coordinate surfaces of the family $u^1 = \text{const}$. The metal must be assumed perfectly conducting; otherwise no TEM wave is possible. The dielectric is homogeneous, isotropic, uncharged, and lossless. At $z = 0$, a sinusoidal emf. is applied between the cylinders, giving the boundary conditions,

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

$$\begin{cases} z = 0 & \text{and} \quad u^1 = a, \quad \varphi = \sqrt{2}\, Ve^{i\omega t}; \\ z = 0 & \text{and} \quad u^1 = b, \quad \varphi = 0. \end{cases} \tag{2}$$

For a TEM wave with such a guide, both $\mathbf{E}$ and $\mathbf{H}$ are in the u^1u^2-plane and

$$\mathbf{E} = \mathbf{a}_1 E_1(u^1, u^2, u^3, t), \qquad \mathbf{H} = \mathbf{a}_2 H_2(u^1, u^2, u^3, t), \tag{3}$$

where $\mathbf{a}_1$ and $\mathbf{a}_2$ are unit vectors. Furthermore, the steady-state a-c. solutions are sinusoidal functions of time and may be written

$$E_1 = \sqrt{2}\, E_1^* e^{i\omega t}, \qquad H_2 = \sqrt{2}\, H_2^* e^{i\omega t}. \tag{4}$$

Here E_1^* and H_2^* are, in general, complex numbers.

The wave equation then reduces to the vector Helmholtz equation,

$$\bigstar E_1^* + \omega^2\epsilon\mu E_1^* = 0, \tag{5}$$

the vector Laplacian being

$$\bigstar \mathbf{E} \equiv \text{grad div } \mathbf{E} - \text{curl curl } \mathbf{E}.$$

Making use of the expression (2) for the vector Laplacian in general curvilinear coordinates, we rewrite Eq. 5 as

$$\frac{1}{\sqrt{g_{11}}}\frac{\partial}{\partial u^1}\left[\frac{1}{\sqrt{g_{11}g_{22}}}\frac{\partial}{\partial u^1}(\sqrt{g_{22}}\,E_1^*)\right] + \frac{1}{\sqrt{g_{22}}}\frac{\partial}{\partial u^2}\left[\frac{1}{\sqrt{g_{11}g_{22}}}\frac{\partial}{\partial u^2}(\sqrt{g_{11}}\,E_1^*)\right]$$
$$+ \frac{1}{\sqrt{g_{22}}}\frac{\partial}{\partial u^3}\left[\sqrt{\frac{g_{22}}{g_{11}}}\frac{\partial}{\partial u^3}(\sqrt{g_{11}}\,E_1^*)\right] + \omega^2\epsilon\mu E_1^* = 0. \tag{5a}$$

But, by Maxwell's equations,

$$\text{div } \mathbf{E}^* = \frac{1}{\sqrt{g}}\left\{\frac{\partial}{\partial u^1}\left(\sqrt{\frac{g}{g_{11}}}\,E_1^*\right) + \frac{\partial}{\partial u^2}\left(\sqrt{\frac{g}{g_{22}}}\,E_2^*\right)\right.$$
$$\left. + \frac{\partial}{\partial u^3}\left(\sqrt{\frac{g}{g_{33}}}\,E_3^*\right)\right\} = 0, \tag{6}$$

$$\text{curl } \mathbf{E}^* = \frac{1}{\sqrt{g}}\left\{\mathbf{a}_1\sqrt{g_{11}}\left[\frac{\partial}{\partial u^2}(\sqrt{g_{33}}\,E_3^*) - \frac{\partial}{\partial u^3}(\sqrt{g_{22}}\,E_2^*)\right]\right.$$
$$+ \mathbf{a}_2\sqrt{g_{22}}\left[\frac{\partial}{\partial u^3}(\sqrt{g_{11}}\,E_1^*) - \frac{\partial}{\partial u^1}(\sqrt{g_{33}}\,E_3^*)\right]$$
$$\left. + \mathbf{a}_3\sqrt{g_{33}}\left[\frac{\partial}{\partial u^1}(\sqrt{g_{22}}\,E_2^*) - \frac{\partial}{\partial u^2}(\sqrt{g_{11}}\,E_1^*)\right]\right\} = -i\omega\mu\mathbf{H}^*. \tag{7}$$

From Eq. 6, since $E_2^* = E_3^* = 0$,

$$\frac{\partial}{\partial u^1}(\sqrt{g_{22}}\,E_1^*) = 0; \tag{8}$$

and from Eq. 7,

$$\left\{\begin{aligned} &\frac{\partial}{\partial u^2}(\sqrt{g_{11}}\,E_1^*) = 0, &(9)\\ &\frac{\partial E_1^*}{\partial z} = -i\omega\mu H_2^*. &(10)\end{aligned}\right.$$

Substitution of Eqs. 8 and 9 into 5*a* gives

$$\frac{\partial^2 E_1^*}{\partial z^2}(u^1, u^2, z) + \omega^2\epsilon\mu E_1^*(u^1, u^2, z) = 0, \tag{11}$$

a solution of which is

$$E_1^* = f(u^1, u^2)e^{-i\omega\sqrt{\epsilon\mu}\,z}. \tag{12}$$

But, according to Eqs. 8 and 9, $\sqrt{g_{22}}\,E_1^*$ is not a function of u^1, and $\sqrt{g_{11}}\,E_1^*$ is not a function of u^2. Thus,

$$\left.\begin{aligned}\sqrt{g_{22}}\,E_1^* &= \sqrt{g_{22}}f(u^1, u^2)e^{-i\omega\sqrt{\epsilon\mu}\,z} = AF_2(u^2)e^{-i\omega\sqrt{\epsilon\mu}\,z},\\ \sqrt{g_{11}}\,E_1^* &= \sqrt{g_{11}}f(u^1, u^2)e^{-i\omega\sqrt{\epsilon\mu}\,z} = AF_1(u^1)e^{-i\omega\sqrt{\epsilon\mu}\,z},\end{aligned}\right\} \tag{13}$$

where F_1 and F_2 are unknown functions, and A is a constant. Dividing the first of these equations by the second, we obtain

$$\frac{F_2(u^2)}{F_1(u^1)} = \sqrt{\frac{g_{22}}{g_{11}}}, \tag{14}$$

which allows the evaluation of F_1 and F_2 from the known metric coefficients. Equation 14 is satisfied by any orthogonal, cylindrical coordinate system; so *there exists an infinite number of coordinate systems allowing TEM waves* (1). Some of these coordinates are described in another paper (3).

The solution of the Helmholtz equation is therefore

$$E_1^* = A\frac{F_1(u^1)}{\sqrt{g_{11}}}e^{-i\omega\sqrt{\epsilon\mu}\,z} = A\frac{F_2(u^2)}{\sqrt{g_{22}}}e^{-i\omega\sqrt{\epsilon\mu}\,z}, \tag{13a}$$

where F_1 and F_2 are obtained from Eq. 14. Also, from Eq. 10,

$$\frac{\partial E_1^*}{\partial z} = -i\omega\mu H_2^* = -i\omega\sqrt{\epsilon\mu}\,E_1^*,$$

or

$$H_2^* = E_1^*/Z_0, \tag{15}$$

where Z_0 is the *intrinsic impedance:*

$$Z_0 = \sqrt{\mu/\epsilon}.$$

The constant A of Eq. 13*a* is evaluated by use of the boundary conditions. At $z = 0$,

$$V = \int_a^b E_1^* ds = A \int_a^b \frac{F_1(u^1)}{\sqrt{g_{11}}} (\sqrt{g_{11}}\, du^1),$$

or

$$A = \frac{V}{\int_a^b F_1(u^1) du^1}. \tag{16}$$

The **E**-vector at any point in the dielectric is

$$\begin{aligned} \mathbf{E} &= \mathbf{a}_1 \sqrt{2}\, E_1^* e^{i\omega t} \\ &= \mathbf{a}_1 \sqrt{2}\, A \frac{F_1(u^1)}{\sqrt{g_{11}}} e^{i\omega(t - \sqrt{\epsilon\mu}\, z)}, \end{aligned} \tag{17}$$

so the wave is propagated always at the velocity of light:

$$v_c = 1/\sqrt{\epsilon\mu}.$$

There is no cut off and there is no attenuation at any frequency. According to Eq. 15, the **H**-vector is always in time phase with the **E**-vector.

The above general conclusions are, of course, well-known; but the simple formulas, Eqs. 14 and 13*a*, are believed to be new. They give a convenient way of writing **E** and **H** by inspection for the TEM wave on any given cylindrical waveguide. The equations are even simpler for a cylindrical coordinate system where $g_{11} = g_{22}$. This category includes (3) all cylindrical systems obtained directly from complex-plane transformations. Then F_1 and F_2 become unity by Eq. 14, and

$$E_1^* = \frac{A}{\sqrt{g_{11}}} e^{-i\omega\sqrt{\epsilon\mu}\, z}, \tag{13b}$$

where A is given by Eq. 16.

SPECIAL CASES

(*I*) As an example, consider the transmission line consisting of two concentric metal cylinders, Fig. 1. Circular-cylinder coordinates

(r, ψ, z) are employed, and

$$g_{11} = 1, \qquad g_{22} = r^2.$$

According to Eq. 14,

$$\frac{F_2(\psi)}{F_1(r)} = \sqrt{\frac{g_{22}}{g_{11}}} = r.$$

Therefore,

$$F_1 = 1/r, \qquad F_2 = 1.$$

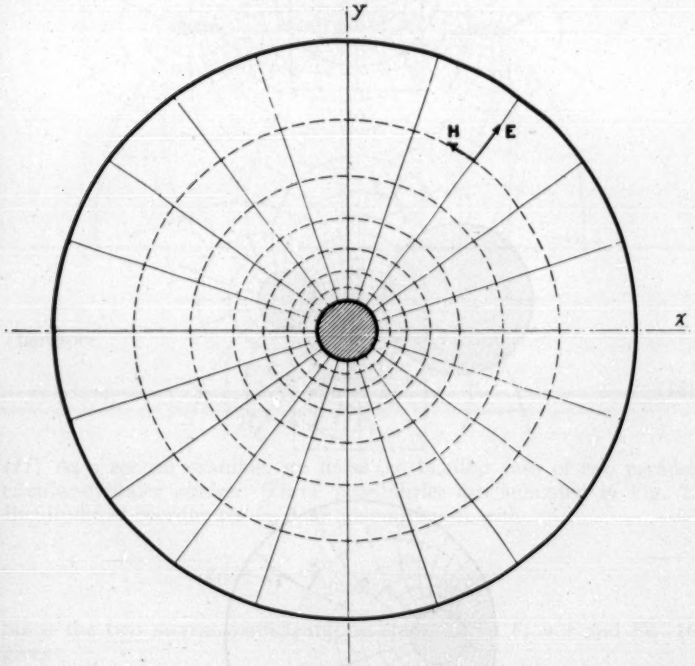
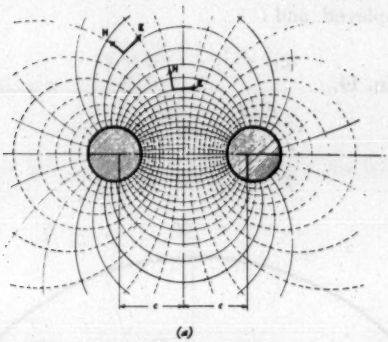


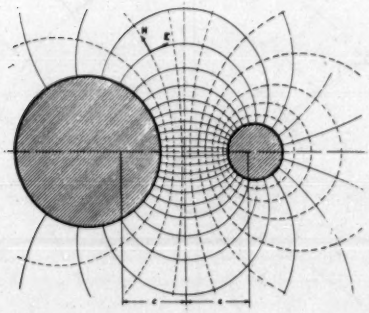
Fig. 1.

The **E**-vector is always radial, and its magnitude varies inversely as the radius. The constant A is obtained from Eq. 16:

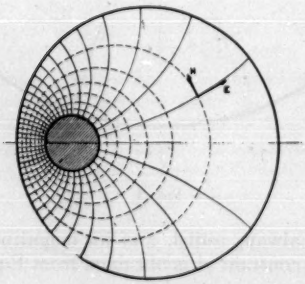
$$A = \frac{V}{\int_a^b \frac{dr}{r}} = \frac{V}{\ln (b/a)}.$$



(a)



(b)



(c)

FIG. 2.

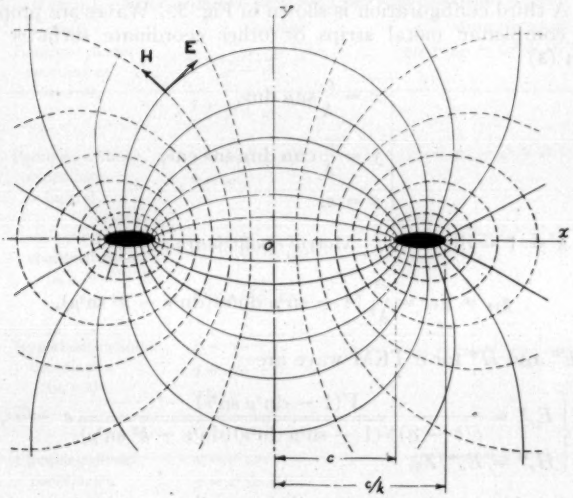


FIG. 3.

Therefore,

$$\begin{cases} E_r^* = \dfrac{V}{r \ln (b/a)} e^{-i\omega\sqrt{\epsilon\mu}\, z}, \\ H_\psi^* = E_r^*/Z_0. \end{cases} \tag{18}$$

(*II*) As a second example, we have the familiar case of two parallel circular-cylinder guides. Three possibilities are indicated in Fig. 2. Bicylindrical coordinates (η, θ, z) are employed, with

$$g_{11} = g_{22} = \frac{c^2}{(\cosh \eta - \cos \theta)^2}.$$

Since the two metric coefficients are equal, $F_1 = F_2 = 1$ and Eq. 16 gives

$$A = \frac{V}{b - a}.$$

Thus, by Eq. 13*b*,

$$\begin{aligned} E_\eta^* &= \frac{V(\cosh \eta - \cos \theta)}{c(b - a)} e^{-i\omega\sqrt{\epsilon\mu}\, z}, \\ H_\theta^* &= E_\eta^*/Z_0. \end{aligned} \tag{19}$$

(*III*) A third configuration is shown in Fig. 3. Waves are propagated along complanar metal strips or other coordinate surfaces of the system (3)

$$\begin{cases} x = \dfrac{c}{\Lambda} \operatorname{sn}\mu \operatorname{dn}\nu, \\ y = \dfrac{c}{\Lambda} \operatorname{cn}\mu \operatorname{dn}\mu \operatorname{sn}\nu \operatorname{cn}\nu, \\ z = z, \end{cases}$$

where $\Lambda = 1 - \operatorname{dn}^2\mu \operatorname{sn}^2\nu$. Metric coefficients are

$$g_{11} = g_{22} = \frac{c^2}{\Lambda^2}(1 - \operatorname{sn}^2\mu \operatorname{dn}^2\nu)(\operatorname{dn}^2\nu - k^2 \operatorname{sn}^2\mu).$$

Thus E^* and H^* for a TEM wave are

$$\begin{cases} E_\mu^* = \dfrac{V(1 - \operatorname{dn}^2\mu \operatorname{sn}^2\nu)}{c(b-a)\sqrt{(1 - \operatorname{sn}^2\mu \operatorname{dn}^2\nu)(\operatorname{dn}^2\nu - k^2 \operatorname{sn}^2\mu)}} e^{-i\omega\sqrt{\epsilon\mu}\, z}, \\ H_\nu^* = E_\mu^*/Z_0. \end{cases} \tag{20}$$

Waveguides conforming to other coordinate systems are handled similarly. A list of metric coefficients for ten coordinate systems (3) is given in Table I.

THE QUASIPOTENTIAL

The general subject of TEM waves is considered in the excellent books of Ramo and Whinnery (4), and Slater (5). In both cases, however, the treatment is in terms of rectangular coordinates, in which boundary conditions for non-rectangular guides are not easily formulated. Ramo and Whinnery separate the term $\partial^2\mathbf{E}^*/\partial z^2$ from the vector Laplacian, obtaining the equation

$$\nabla^2\mathbf{E}^*(u^1, u^2) = 0. \tag{21}$$

This looks so much like Laplace's equation

$$\nabla^2\varphi = 0,$$

that they conclude, "the field distribution in the transverse plane is exactly a static distribution." The difficulty is that Eq. 21 refers to the *vector* Laplacian, while Laplace's equation applies to the *scalar* Laplacian (2). Even when the vector possesses one component only, the two Laplacians are still different, except in special cases.

Slater introduces a sort of *ad hoc* potential φ, though the ordinary scalar potential of electrostatics is inapplicable. Slater's treatment can

TABLE I.—*Metric Coefficients for a Few Cylindrical Coordinate Systems.*

No.	Name	Equations	Metric Coefficients
1	Tangent-cylinder coordinates (μ, ν, z)	$x = \frac{\mu}{\mu^2 + \nu^2}$, $y = \frac{\nu}{\mu^2 + \nu^2}$	$g_{11} = g_{22} = (\mu^2 + \nu^2)^{-2}$
2	Parabolic-cylinder coordinates (μ, ν, z)	$x = \frac{1}{2}(\mu^2 - \nu^2)$, $y = \mu\nu$.	$g_{11} = g_{22} = \mu^2 + \nu^2$
3	Cardioid-cylinder coordinates (μ, ν, z)	$x = \frac{1}{2}\frac{\mu^2 - \nu^2}{(\mu^2 + \nu^2)^2}$, $y = \frac{\mu\nu}{(\mu^2 + \nu^2)^2}$.	$g_{11} = g_{22} = (\mu^2 + \nu^2)^{-2}$
4	Hyperbolic-cylinder coordinates (μ, ν, z)	$x = \sqrt{\rho + \mu}$, $y = \sqrt{\rho - \mu}$, where $\rho = +\sqrt{\mu^2 + \nu^2}$.	$g_{11} = g_{22} = \frac{1}{2\sqrt{\mu^2 + \nu^2}}$
5	Circular-cylinder coordinates (η, ψ, z)	$x = e^{\eta}\cos\psi$, $y = e^{\eta}\sin\psi$.	$g_{11} = g_{22} = e^{2\eta}$
6	Bicylindrical coordinates (η, θ, z)	$x = \frac{c\sinh\eta}{\cosh\eta - \cos\theta}$, $y = \frac{c\sin\theta}{\cosh\eta - \cos\theta}$.	$g_{11} = g_{22} = \frac{c^2}{(\cosh\eta - \cos\theta)^2}$
7	Elliptic-cylinder coordinates (η, θ, z)	$x = c\cosh\eta\cos\theta$, $y = c\sinh\eta\sin\theta$.	$g_{11} = g_{22} = c^2(\cosh^2\eta - \cos^2\theta)$
8	(η, θ, z)	$x = \frac{c\cosh\eta\cos\theta}{\cosh^2\eta - \sin^2\theta}$, $y = \frac{c\sinh\eta\sin\theta}{\cosh^2\eta - \sin^2\theta}$.	$g_{11} = g_{22} = \frac{c^2(\cosh^2\eta - \cos^2\theta)}{(\cosh^2\eta - \sin^2\theta)^2}$
9	(μ, ν, z)	$x = \frac{c}{\Lambda}\,\mathrm{sn}(\mu, k)\cdot\mathrm{dn}(\nu, k')$, $y = \frac{c}{\Lambda}\,\mathrm{cn}\mu\,\mathrm{dn}\mu\,\mathrm{sn}\nu\,\mathrm{cn}\nu$, where $\Lambda = 1 - \mathrm{dn}^2\mu\,\mathrm{sn}^2\nu$.	$g_{11} = g_{22} = \frac{c^2}{\Lambda^2}(1 - \mathrm{sn}^2\mu\,\mathrm{dn}^2\nu)\cdot(\mathrm{dn}^2\nu - k^2\,\mathrm{sn}^2\mu)$
10	(μ, ν, z)	$x = \frac{c}{\Lambda}\,\mathrm{cn}(\mu, k)\cdot\mathrm{cn}(\nu, k')$, $y = \frac{c}{\Lambda}\,\mathrm{sn}\mu\,\mathrm{dn}\mu\,\mathrm{sn}\nu\,\mathrm{dn}\nu$.	$g_{11} = g_{22} = \frac{c^2}{\Lambda^2}(\mathrm{sn}^2\nu + \mathrm{sn}^2\mu\,\mathrm{cn}^2\nu)\cdot(\mathrm{dn}^2\nu - k^2\,\mathrm{sn}^2\mu)$

be brought into better alignment with established mathematics by using the familiar *quasipotential* Φ. The necessary and sufficient condition (6) for the existence of a quasipotential is

$$\mathbf{E}^* \cdot \operatorname{curl} \mathbf{E}^* = 0. \tag{22}$$

The quasipotential is defined by the relation,

$$\mathbf{E}^* = -\frac{1}{\zeta(u^1, u^2, u^3)} \nabla\Phi, \tag{23}$$

where ζ is a weighting function and where the quasipotential satisfies Laplace's equation,

$$\nabla^2\Phi = 0. \tag{24}$$

For a TEM wave,

$$\mathbf{E}^* = \mathbf{a}_1 E_1^*(u^1, u^2, u^3);$$

and, from Eq. 7,

$$\operatorname{curl} \mathbf{E}^* = \mathbf{a}_2 \frac{\partial E_1^*}{\partial z} - \mathbf{a}_3 \frac{1}{\sqrt{g}} \frac{\partial}{\partial u^2} (\sqrt{g_{11}}\, E_1^*). \tag{7a}$$

Since $\mathbf{E}^*$ is in the u^1-direction while curl $\mathbf{E}^*$ lies in a u^2u^3-surface, the two are always orthogonal and Eq. 22 is satisfied. Therefore, *a quasipotential always exists for a TEM wave.*

The $\mathbf{E}^*$-vector has only one component, so Eq. 23 becomes

$$E_1^* = -\frac{1}{\zeta\sqrt{g_{11}}} \frac{\partial\Phi}{\partial u^1}, \qquad \frac{\partial\Phi}{\partial u^2} = \frac{\partial\Phi}{\partial u^3} = 0. \tag{25}$$

Laplace's equation in generalized coordinates is

$$\sum_{i=1}^{3} \frac{\partial}{\partial u^i} \left(\frac{\sqrt{g}}{g_{ii}} \frac{\partial\Phi}{\partial u^i} \right) = 0; \tag{24a}$$

and by use of Eq. 25,

$$\frac{d}{du^1} \left(\sqrt{\frac{g_{22}}{g_{11}}} \frac{d\Phi}{du^1} \right) = 0. \tag{24b}$$

To utilize this method for TEM waves with any cylindrical coordinate system, we obtain the quasipotential by solving Eq. 24*b*. The quasipotential is constant over a metal surface, just as the scalar potential is constant in electrostatics; but E_1^* is known to vary in the z-direction as $e^{-i\omega\sqrt{\epsilon\mu}\, z}$. Thus, *for any waveguide with TEM wave,*

$$\frac{1}{\zeta} = e^{-i\omega\sqrt{\epsilon\mu}\, z}. \tag{26}$$

The electric field strength E_1^* is, therefore,

$$E_1^* = -\frac{1}{\sqrt{g_{11}}}\frac{d\Phi}{du^1}e^{-i\omega\sqrt{\epsilon\mu}\,z}. \tag{25a}$$

In circular-cylinder coordinates, for example, Eq. 24*b* becomes

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r}\frac{d\Phi}{dr} = 0,$$

whose complete solution is

$$\Phi = A \ln r + B.$$

Boundary conditions are

$$\begin{cases} r = a, & \Phi = V; \\ r = b, & \Phi = 0. \end{cases}$$

These conditions evaluate A and B, giving as solution,

$$\Phi = \frac{V}{\ln (b/a)} \ln (b/r). \tag{27}$$

From Eq. 25*a*,

$$E_r^* = -\frac{d\Phi}{dr}e^{-i\omega\sqrt{\epsilon\mu}\,z} = \frac{V}{r \ln (b/a)}e^{-i\omega\sqrt{\epsilon\mu}\,z}. \tag{18}$$

Similarly, for bicylindrical guides, Eq. 24*b* is

$$\frac{d^2\Phi}{d\eta^2} = 0,$$

with solution

$$\Phi = \frac{V}{b - a}(b - \eta). \tag{28}$$

Since

$$g_{11} = g_{22} = \frac{c^2}{(\cosh \eta - \cos \theta)^2},$$

$$E_\eta^* = \frac{1}{\sqrt{g_{11}}}\frac{V}{(b - a)}e^{-i\omega\sqrt{\epsilon\mu}\,z} = \frac{V(\cosh \eta - \cos \theta)}{c(b - a)}e^{-i\omega\sqrt{\epsilon\mu}\,z}. \tag{19}$$

An infinite number of other configurations can be handled in the same way.

The foregoing pages have outlined two very simple ways of treating the theory of TEM waves guided by cylindrical conductors. The first

method works directly with the **E**-vector and the vector Laplacian; the second introduces a scalar quasipotential, from which the **E**-vector is obtained. Both methods are rigorously correct and constitute a logical advance over previous general theory of TEM waves.

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ON TURBULENT FLOW IN CIRCULAR PIPE

BY

S. I. PAI[1]

ABSTRACT

The Reynolds equations of motion of turbulent flow of an incompressible fluid have been studied for turbulent flow in circular pipe. The number of these equations is finally reduced to two. One of these consists of mean velocity and correlation between radial and axial turbulent velocity fluctuations $\overline{u'w'}$ only. The other consists of the mean pressure, the radial and the circumferential turbulent velocity intensity only. Some conclusions about the mean pressure distribution and turbulent fluctuations are drawn. The mean velocity distribution and the correlation $\overline{u'w'}$ can be expressed in a form of polynomial of the radial distance with the ratio of shearing stress on the wall to that of the corresponding laminar flow of the same maximum velocity as a parameter. These expressions hold true all the way across the tube, that is, both the turbulent region and viscous layer including the laminar sublayer. The mean velocity distribution has been compared with Nikuradse's experimental data to turbulent flow in a circular pipe.

It also shows that the logarithmic mean velocity distribution is not a rigorous solution of Reynolds equations.

I. INTRODUCTION

Actual flows at large Reynolds number are characterized by a peculiar phenomenon known as "turbulence." In such flows, the apparently steady motion of fluids is only steady insofar as the temporal mean values of the velocities and the pressure are concerned; in reality velocities and pressures are subjected in irregular fluctuations. The essential characteristic of turbulent motion is that the turbulent fluctuations are random in nature. Hence the final and logical solution of turbulence problem requires the application of methods of statistical mechanics. It was Osborne Reynolds (1)[2] who first introduced elementary statistical motions into the consideration of the turbulence problem. The motion of a fluid as usually considered in hydrodynamics is obtained by considering the means of the molecular motions, and according to the kinetic theory of gases, for example, the velocity components, pressure, density, temperature, etc. of a gas in motion may be defined and approximately satisfy the Navier-Stokes equations. In turbulent motion, however, even if the fluid is regarded as a continuum, we still have to deal with turbulent velocity fluctuations superimposed on a mean motion. We may write

$$u_i = \bar{u}_i + u_i', \tag{1}$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

where u_i is the ith component of the total fluid velocity, $\bar{u}_i$ is the ith mean velocity component, and u_i' is the ith component of the turbulent fluctuating velocity. If we substitute u_i from Eq. 1 into the Navier-Stokes equations of motion for an incompressible fluid and use the rules that we say derive for taking means, we obtain a set of equations known as *Reynolds equations* which differ from Navier-Stokes equations only in the presence of additional terms called the Reynolds stresses or eddy stresses. They form a tensor with components $-\rho\overline{u_i'u_j'}$, the eddy normal stress components being $-\rho\overline{u_i'^2}$ and the eddy shearing stress components being $-\rho\overline{u_i'u_j'}$ $(i \neq j)$, where ρ is the density of the fluid. These stresses are the representation of the rate of transfer of momentum across the corresponding surfaces due to the turbulent velocity fluctuations.

The solutions of Reynolds equations will represent properly the turbulent flow. However there are only four Reynolds equations for ten unknowns: the mean pressure, three mean velocity components and six Reynolds stresses. In general, the Reynolds equations are not sufficient to determine these unknowns.

The Reynolds equations have been extensively studied for the simplest type of turbulence, isotropic turbulence, by Taylor (2), von Kármán (3), Heisenberg (4), Kolmogoroff (5), and others. Excellent reviews for the present status of the statistical theory of isotropic turbulence are given in references 6, 7, and 8. Even in such a simple case, it is not possible to solve this problem completely because the number of equations is less than the number of unknowns. Our knowledge about the statistical theory of nonhomogeneous turbulence is even more meager (3, 9, 10). There is indeed much to be done before practical engineers can make real use of the statistical theory of turbulence.

As far as the mean velocity distribution is concerned, some semi-empirical theories of fully developed turbulent flow have been successfully developed. The best known theory among them is Prandtl's mixing length theory (11, 12, 13). The famous von Kármán's logarithmic velocity distribution is derived from the mixing length theory (14). Some critical discussions about the logarithmic velocity profiles have been made (15, 16, 17). Even though Prandtl's mixing length theory had successfully predicted the mean velocity distribution in turbulent flow, it is now generally believed to have serious limitations and inconsistencies (18). The mixing length deduced from measurements of the mean velocity is of the same order of magnitude of the dimension of the mean flow, but in theory, it should be very small. The assumptions on the distribution of mixing length are in general not true as revealed from hot-wire anemometer measurements. The boundary conditions at the wall are not satisfied, thus the logarithmic velocity distribution is not a rigorous solution of Reynolds equations.

Usually one can only say that the results of the mixing length theory do not hold true for viscous layers and laminar sublayers which are near the wall. Furthermore it fails to predict anything about turbulent fluctuations.

In order to understand more thoroughly the mechanism of turbulent flow, it is better to study directly the Reynolds equations without introducing the concept of mixing length. Professor Kampe de Feriet (19) recently studied the Reynolds equations for turbulent flow of an incompressible fluid between two fixed parallel walls, that is, the so-called plane Poiseuille flow in reference (19). The analysis has been extended by the present author (20) such that rigorous solutions for mean velocity distributions between parallel plates, both plane Poiseuille flow and plane Couette flow, have been found which hold true for the whole region between the plates. The plane Poiseuille case has been checked experimentally, with fairly good results, for turbulent flow in a channel.

This paper studies the turbulent flow in a circular pipe by an analysis similar to that given by the author in reference 20. A rigorous solution of Reynolds equation for this case is also obtained. The theoretical results are checked with the experimental data.

It also shows that the logarithmic velocity distribution is not a rigorous solution of the Reynolds equations in the present case.

II. STATEMENT OF THE PROBLEM

Consider the turbulent flow of an incompressible fluid in a circular pipe of radius a. The cylindrical coordinates r, θ, z are shown in Fig. 1,

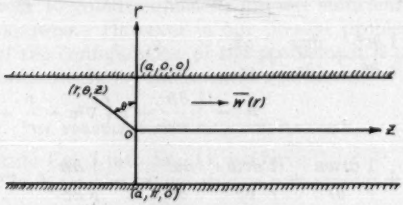

FIG. 1. Flow in a circular pipe.

where $z=0$ is arbitrarily chosen, and z is extended to both $+\infty$ and $-\infty$.

We assume that the mean flow is steady and axially symmetrical, that is,

$$\frac{\partial \bar{f}}{\partial t} = 0, \qquad \frac{\partial \bar{f}}{\partial \theta} = 0, \qquad \therefore \quad \bar{f} = \bar{f}(r, z), \tag{2}$$

where $\bar{f}$ may represent any quantity of the mean flow such as mean velocity components or mean pressure etc.

The instantaneous velocity components of the flow are u, v, and w in the direction of r, θ and z, respectively. We assume that the mean flow has velocity components along the z-axis only, that is,

$$\bar{w} = \bar{w}(r, z), \qquad \bar{u} = \bar{v} = 0. \tag{3}$$

The turbulent velocity components along r, θ, and z directions, respectively, are

$$u' = u, \qquad v' = v, \qquad w' = w - \bar{w}. \tag{4}$$

In the Reynolds equations we have the following mean values of turbulent fluctuations

$$\overline{u'^2}(r), \qquad \overline{v'^2}(r), \qquad \overline{w'^2}(r), \qquad \overline{u'v'}(r), \qquad \overline{v'w'}(r), \qquad \overline{w'u'}(r). \tag{5}$$

Here we assume that these mean values of turbulent fluctuations are functions of r only. This is due to the fact that we consider a problem which has the same statistical properties at all planes z = constant.

III. FUNDAMENTAL EQUATIONS

We assume that the instantaneous velocity components of the flow satisfy the Navier-Stokes equations which in cylindrical coordinates (11) are as follows:

$$\frac{\partial u}{\partial t} + \frac{1}{r}\frac{\partial ru^2}{\partial r} + \frac{1}{r}\frac{\partial vu}{\partial \theta} + \frac{\partial wu}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right), \tag{6}$$

$$\frac{\partial v}{\partial t} + \frac{1}{r}\frac{\partial ruv}{\partial r} + \frac{1}{r}\frac{\partial v^2}{\partial \theta} + \frac{\partial vw}{\partial z} + \frac{uv}{r} = -\frac{1}{\rho}\frac{\partial p}{r\partial \theta} + \nu\left(\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right), \tag{7}$$

$$\frac{\partial w}{\partial t} + \frac{1}{r}\frac{\partial rwu}{\partial r} + \frac{1}{r}\frac{\partial vw}{\partial \theta} + \frac{\partial w^2}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w, \tag{8}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}.$$

The equation of continuity is

$$\frac{1}{r}\frac{\partial ru}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0. \tag{9}$$

Substituting

$$p = \bar{p} + p', \qquad u = \bar{u} + u', \qquad v = \bar{v} + v', \qquad w = \bar{w} + w', \tag{10}$$

into Eqs. 6 to 9 and averaging, we have

$$\frac{\partial \bar{u}}{\partial t} + \frac{1}{r}\frac{\partial r\bar{u}^2}{\partial r} + \frac{1}{r}\frac{\partial \bar{v}\bar{u}}{\partial \theta} + \frac{\partial \bar{w}\bar{u}}{\partial z} - \frac{\bar{v}^2}{r} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial r} + \nu\left(\nabla^2\bar{u} - \frac{\bar{u}}{r^2} - \frac{2}{r^2}\frac{\partial \bar{v}}{\partial \theta}\right) - \frac{1}{r}\frac{\partial r\overline{u'^2}}{\partial r} - \frac{1}{r}\frac{\partial \overline{v'u'}}{\partial \theta} - \frac{\partial \overline{u'w'}}{\partial z} + \frac{\overline{v'^2}}{r}, \quad (11)$$

$$\frac{\partial \bar{v}}{\partial t} + \frac{1}{r}\frac{\partial r\bar{v}\bar{u}}{\partial r} + \frac{1}{r}\frac{\partial \bar{v}^2}{\partial \theta} + \frac{\partial \bar{w}\bar{v}}{\partial z} + \frac{\bar{u}\bar{v}}{r} = -\frac{1}{\rho}\frac{\partial \bar{p}}{r\partial \theta} + \nu\left(\nabla^2\bar{v} + \frac{2}{r^2}\frac{\partial \bar{u}}{\partial \theta} - \frac{\bar{v}}{r^2}\right) - \frac{1}{r}\frac{\partial r\overline{u'v'}}{\partial r} - \frac{1}{r}\frac{\partial \overline{v'^2}}{\partial \theta} - \frac{\partial \overline{w'v'}}{\partial z} - \frac{\overline{u'v'}}{r}, \quad (12)$$

$$\frac{\partial \bar{w}}{\partial t} + \frac{1}{r}\frac{\partial r\bar{w}\bar{u}}{\partial r} + \frac{1}{r}\frac{\partial \bar{w}\bar{v}}{\partial \theta} + \frac{\partial \bar{w}^2}{\partial z} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z} + \nu\nabla^2\bar{w} - \frac{1}{r}\frac{\partial r\overline{w'u'}}{\partial r} - \frac{1}{r}\frac{\partial \overline{w'v'}}{\partial \theta} - \frac{\partial \overline{w'^2}}{\partial z}, \quad (13)$$

$$\frac{1}{r}\frac{\partial r\bar{u}}{\partial r} + \frac{1}{r}\frac{\partial \bar{v}}{\partial \theta} + \frac{\partial \bar{w}}{\partial z} = 0. \quad (14)$$

Equations 11 to 14 are known as Reynolds equations in cylindrical coordinates, which are the fundamental equations for turbulent flow of incompressible fluids. There are ten unknowns $\bar{p}(1)$, $\bar{u}$, $\bar{v}$, $\bar{w}(3)$, $\overline{u'v'}$, etc. (6). The four Reynolds equations are not sufficient in general to find the ten unknowns. However in our present problem, because of the simplicity of the configuration of the problem, it is possible to get some information from these fundamental equations.

IV. TURBULENT FLOW IN A CIRCULAR PIPE

If we substitute Eq. 3 into Eq. 14, we have

$$\frac{\partial \bar{w}}{\partial z} = 0,$$

or

$$\bar{w} = \bar{w}(r). \quad (15)$$

We see that the mean velocity $\bar{w}$ is a function of r only.

Substituting the values of Eqs. 2, 3, and 5 into Eq. 12, we have

$$\frac{1}{r}\frac{d r\overline{u'v'}}{dr} + \frac{\overline{u'v'}}{r} = 0. \quad (16)$$

Integrating Eq. 16 we have

$$\overline{u'v'} = \frac{\text{constant}}{r^2}. \tag{17}$$

But on the wall of the tube $r = a$, $\overline{u'v'} = 0$, hence the constant in Eq. 17 must be zero. Thus

$$\overline{u'v'} = 0,$$

for the whole velocity field. In this type of flow, the correlation of two transverse turbulent fluctuating velocity components is zero for the entire flow field.

Equations 11 and 13 become

$$\frac{1}{\rho}\frac{\partial \bar{p}}{\partial r} + \frac{1}{r}\frac{\partial r\overline{u'^2}}{\partial r} - \frac{\overline{v'^2}}{r} = 0, \tag{18}$$

$$\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z} - \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + \frac{1}{r}\frac{\partial r\overline{w'u'}}{\partial r} = 0. \tag{19}$$

Now we have five unknowns $w(r)$, $\bar{p}(r, z)$, $\overline{w'u'}(r)$, $\overline{u'^2}(r)$ and $\overline{v'^2}(r)$, but only two Eqs. 18 and 19. We cannot solve this problem completely, but it is possible to find some relations among these five unknown quantities.

V. NON-DIMENSIONAL FORMS OF FUNDAMENTAL EQUATIONS

We define the non-dimensional coordinates as follows:

$$\xi = \frac{z}{a}, \qquad \eta = \frac{r}{a}. \tag{20}$$

Let τ_0 be the shearing stress on the wall of the tube, that is, at $\eta = 1$. τ_0 is a constant or rather a parameter in the present problem. We define a reference velocity

$$U^* = \sqrt{\frac{\tau_0}{\rho}}, \tag{21}$$

and the corresponding Reynolds number R^*

$$R^* = \frac{U^* a}{\nu}. \tag{22}$$

The non-dimensional mean velocity is defined as

$$V = \frac{w}{U^*} = V(\eta, R^*). \tag{23}$$

The average of the mean velocity over any section ξ = constant is U_0, that is,

$$U_0 = \frac{2}{a^2}\int_0^a wrdr; \qquad \frac{U_0}{U^*} = 2\int_0^1 V\eta d\eta. \tag{24}$$

The corresponding Reynolds number is

$$R_0 = \frac{U_0 a}{\nu}. \tag{25}$$

The coefficient of friction of the pipe is defined as

$$C_f = \frac{\tau_0}{\rho U_0^2/2}. \tag{26}$$

Hence

$$\sqrt{\frac{C_f}{2}} = \frac{U^*}{U_0} = \frac{R^*}{R_0}. \tag{27}$$

The non-dimensional pressure is defined as

$$\bar{\omega} = \frac{\bar{p} - \bar{p}_0}{\rho U^{*2}} = \bar{\omega}(\xi, \eta, R^*), \tag{28}$$

where $\bar{p}_0$ is a reference pressure which may be taken as the mean pressure at the point $\xi = 0$, $\eta = 1$.

The non-dimensional forms of turbulent fluctuations are

$$y(\eta, R^*) = \frac{\overline{u'w'}}{U^{*2}}, \tag{29}$$

$$\sigma_1(\eta, R^*) = \frac{\overline{u'^2}}{U^{*2}}, \tag{30}$$

$$\sigma_2(\eta, R^*) = \frac{\overline{v'^2}}{U^{*2}}. \tag{31}$$

In non-dimensional form, Eqs. 18 and 19 become

$$\frac{\partial\bar{\omega}}{\partial\eta} + \frac{1}{\eta}\frac{d\eta\sigma_1}{\partial\eta} - \frac{\sigma_2}{\eta} = 0, \tag{32}$$

$$\frac{\partial\bar{\omega}}{\partial\xi} - \frac{1}{R^*}\left(\frac{d^2V}{d\eta^2} + \frac{1}{\eta}\frac{dV}{d\eta}\right) + \frac{1}{\eta}\frac{d\eta y}{d\eta} = 0. \tag{33}$$

The boundary conditions are

$$V(1, R^*) = 0, \tag{34}$$

$$\frac{dV}{d\eta} = 0, \qquad \text{at} \qquad \eta = 0, \tag{35}$$

$$y(1, R^*) = 0, \tag{36}$$

$$\sigma_1(1, R^*) = 0, \tag{37}$$

$$\sigma_2(1, R^*) = 0, \tag{38}$$

and

$$\tilde{\omega}(0, 1, R^*) = 0. \tag{39}$$

Equation 39 is obtained by the proper choice of $\tilde{p}_0$.

Integration of Eqs. 32 and 33 gives

$$\tilde{\omega}(\xi, \eta, R^*) + \sigma_1(\eta, R^*) + \int_1^\eta \frac{\sigma_1 - \sigma_2}{\eta}\, d\eta = A\xi + C, \tag{40}$$

$$\frac{1}{R^*}\,\eta\,\frac{dV}{d\eta} - \eta y = \frac{A\eta^2}{2} + B, \tag{41}$$

where A, B, and C are constants. From Eqs. 39 and 40 we have $C = 0$. Hence the mean pressure $\tilde{\omega}$ becomes

$$\tilde{\omega}(\xi, \eta, R^*) = A\xi + P(\eta, R^*), \tag{42}$$

where $P(\eta, R^*)$ is a function of η and R^* only, but independent of ξ.

The shearing stress on the wall of the pipe in non-dimensional form is

$$R^* = -\frac{dV}{d\eta}, \qquad \text{at} \qquad \eta = 1. \tag{43}$$

At $\eta = 0$, both y and $dV/d\eta$ are finite, hence $B = 0$. Substituting, then, (43) in (41), we have

$$A = -2, \qquad B = 0. \tag{44}$$

Equations 40 and 41 become

$$\tilde{\omega} + \sigma_1 + \int_1^\eta \frac{\sigma_1 - \sigma_2}{\eta}\, d\eta = -2\xi, \tag{45}$$

$$-\frac{1}{R^*}\frac{dV}{d\eta} + y = \eta. \tag{46}$$

It is interesting to note that the five unknown quantities are divided into two groups V and y in one group and $\bar{\omega}$, σ_1 and σ_2 in the other group. The correlation $\overline{u'w'}$ influences only the mean velocity profile, while the transverse turbulent velocity intensities $\overline{u'^2}$ and $\overline{v'^2}$ affect only the mean pressure distribution.

VI. MEAN PRESSURE DISTRIBUTION

From Eq. 45, two practically interesting conclusions may be drawn for the mean pressure:

(*a*) Since $\sigma_1(\eta)$ and $\sigma_2(\eta)$ are independent of ξ, the mean pressure $\bar{\omega}$ decreases linearly with increase of ξ in the same manner as in laminar flow.

(*b*) From experimental data of Fage,[3] we have that near the wall $\eta \cong 1$, $\sigma_2 > \sigma_1$ and near the center $\eta = 0$, $\sigma_1 = \sigma_2$. Then the mean pressure in the fluid given by Eq. 45 will be greater than or equal to the mean pressure on the wall if we substitute these experimental σ_1 and σ_2 in Eq. 45. This point differs entirely from the turbulent flow in a two-dimensional channel where the mean pressure in the fluid is less than or equal to that on the wall (20).

On the center of the pipe, from Eq. 45, we expect that $\sigma_1 = \sigma_2$, otherwise the mean pressure would be infinite. In Fage's experimental data, one sees that σ_1 is always equal to σ_2 for various Reynolds numbers, at the center of the pipe.

VII. MEAN VELOCITY DISTRIBUTION

Because the mean flow is axially symmetrical, we have the boundary condition (35). Substituting Eq. 35 into Eq. 46, we have

$$y(0, R^*) = 0. \tag{47}$$

The correlation $\overline{u'w'}$ is zero on the axis of the pipe.

Integration of Eq. 46 gives

$$V(\eta, R^*) = \frac{R^*}{2}(1 - \eta^2) - R^* \int_\eta^1 y d\eta. \tag{48}$$

If $y(\eta, R^*) \equiv 0$, Eq. 48 reduces to the velocity profile of laminar Poiseuille flow.

The maximum mean velocity which occurs on the axis of the pipe is given by the following formula

$$\frac{U_m}{U^*} = V(0, R^*) = \frac{R^*}{2} - R^* \int_0^1 y d\eta. \tag{49}$$

It is advisable here to find an expression of velocity distribution which satisfies the differential equation (46), the boundary conditions

[3] Figure 131 (p. 394) of Reference 11.

(34) and (36) and also the experimental results. Thus the velocity distribution will be an exact solution of the Reynolds equations. It should be noticed that because in this problem the number of unknowns is more than the number of the differential equations the solution will not be unique. But if a suitable velocity distribution may be found, it will give us some information about the general characteristics of the velocity profile in the central portion of the pipe as well as that near the wall. In the latter case, it will give us some more rigorous results to compare the ordinary semi-empirical concept of laminar sublayer. It was found that the solution of $V(\eta, R^*)$ may be written in the following form

$$V = \frac{U_m}{U^*}[1 + a_2\eta^2 + a_{2n}\eta^{2n}], \tag{50}$$

where n is an integer larger than or equal to 2. In order to satisfy the boundary conditions (34) and (36) and Eq. 46, we have

$$a_2 = \frac{s - n}{n - 1}, \qquad a_{2n} = \frac{1 - s}{n - 1}, \tag{51}$$

where

$$s = \frac{R^*U^*}{2U_m} = \frac{\tau_0}{\mu\dfrac{2U_m}{a}} = \frac{\tau_0}{\tau_l}, \tag{52}$$

that is, s is the ratio of the shearing stress τ_0 on the wall of the pipe for the turbulent flow to the corresponding shearing stress τ_l on the wall for the laminar flow with the same maximum velocity U_m.

Hence the mean velocity profile is

$$V(\eta, R^*) = \frac{U_m}{U^*}\left[1 + \frac{s - n}{n - 1}\eta^2 + \frac{1 - s}{n - 1}\eta^{2n}\right], \tag{53}$$

and the correlation y is

$$y(\eta, R^*) = \frac{n(s - 1)}{s(n - 1)}\eta[1 - \eta^{2(n-1)}]. \tag{54}$$

If $s = 1$, Eq. 53 reduces automatically to the case of laminar flow and Eq. 54 is identical to zero. In turbulent flow s is larger than 1. The value of n has to be determined from experimental data. From Nikuradse's measurements (21) of turbulent flow in circular pipe, for the case where Reynolds number $2R_0 = 2U_0a/\nu = 3240 \times 10^3$, the mean velocity distribution is found as follows:

$$V_m(\eta, R^*) = 1 - 0.204\eta^2 - 0.796\eta^{32}, \tag{55}$$

where $V_m = v/U_m$, $n = 16$ and $s = 12.94$. The theoretical curve of (55) and Nikuradse's data are plotted in Fig. 2. In the major portion of the pipe, the theoretical curve checks fairly well with the experimental data but near the wall of the pipe, the experimental data are much higher than the theoretical curve. The present author believes that Nikuradse's data near the wall are too high to be true, for instance on the wall, instead of zero velocity, Nikuradse found that $V_m(1) = 0.546$, at $R_0 = 1620 \times 10^3$ which is evidently not quite correct.

In order to see the curvature of the mean velocity distribution near the wall, if we take Nikuradse's experimental data on the wall $\eta = 1$

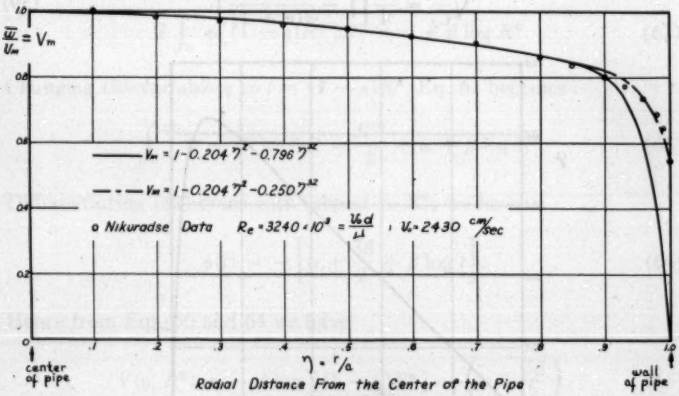
1.0
$\frac{u}{U_m} = V_m$
0.8
0.6
0.4
0.2
0
$V_m = 1 - 0.204\eta^2 - 0.796\eta^{32}$
$V_m = 1 - 0.204\eta^2 - 0.250\eta^{32}$
o Nikuradse Data $R_e = 3240 \times 10^3 = \frac{U_0 d}{\mu}$, $U_0 = 2430$ cm/sec
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
center of pipe
$\eta = r/a$
wall of pipe
Radial Distance From the Center of the Pipe

FIG. 2. Velocity distribution in a circular pipe.

as one of the boundary conditions, we can adjust the coefficient of Eq. 55. The suitable mean velocity distribution is then, at $R_0 = 1620 \times 10^3$,

$$V_m(\eta) = 1 - 0.204\eta^2 - 0.250\eta^{32}. \tag{56}$$

Equation 56 is also plotted in Fig. 2. It gives a much better agreement with experimental data and it shows that near the wall the mean velocity distribution behaves like $1 - a_2\eta^2 - a_{2n}\eta^{32}$. In order to check the theoretical curve (55) with experimental data, the author believes that refined measurements, such as those by hot wire anemometer, should be measured near the wall, in order to determine accurately the mean velocity distribution near the wall to compare with the theoretical curve.

The correlation y corresponding to Eq. 55 is, at $R_0 = 1620 \times 10^3$,

$$y(\eta) = 0.9835\eta(1 - \eta^{30}). \tag{57}$$

Equation 57 is plotted in Fig. 3. Since there are no experimental data available for circular pipe, it is not checked with experimental data.

The coefficient of friction C_f is given by the following formula from Eqs. 24, 27, and 48

$$\sqrt{\frac{2}{C_f}} = \frac{R^*}{4} - R^* \int_0^1 \eta y d\eta. \tag{58}$$

Substituting the value of y of (54) into (58) we have

$$\sqrt{\frac{2}{C_f}} = \frac{R^*}{4}\left[1 - \frac{n(s-1)}{s(n+1)}\right]. \tag{59}$$

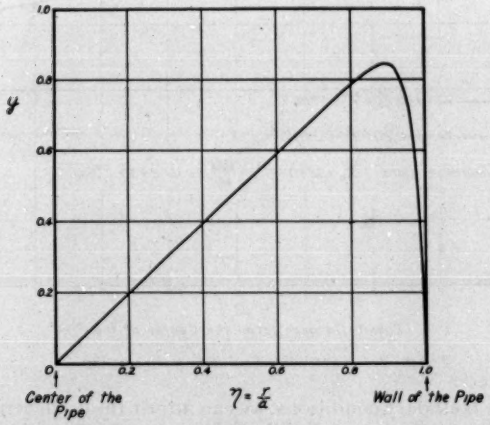

FIG. 3. Correlation $y = \frac{\overline{u'w'}}{U^{*2}}$ distribution in a circular pipe.

VIII. LOGARITHMIC VELOCITY DISTRIBUTION

Similar to the case of turbulent flow in a channel (19), the necessary and sufficient conditions for the existence of logarithmic velocity distribution of von Kármán in a circular pipe are as follows:[4]

(*a*) There exists a universal profile with abscissa $\delta U^*/\nu$, where δ is the distance from the wall, and ordinate w/U^* as obtained experimentally by Nikuradse (21). In the present notation, we have

$$V(\eta, R^*) = \phi[(1-\eta)R^*], \qquad 0 \leq \eta \leq 1. \tag{60}$$

[4] This derivation is also similar to those given in References 15 and 16.

(*b*) The average mean velocity over a section is given by the following formula

$$2\int_0^1 V\eta d\eta = \alpha + \beta \log R^*, \tag{61}$$

where α and β are constant. This formula is sometimes known as Stanton's Law. It is interesting to note that in derivation of the logarithmic velocity distribution for turbulent flow in channel, a different form of this Stanton's Law (61) should be used (19).

Substituting Eq. 60 into Eq. 61, we have

$$2\int_0^1 \phi[(1-\eta)R^*]\eta d\eta = \alpha + \beta \log R^*. \tag{62}$$

Changing the variable η to $t = (1-\eta)R^*$, Eq. 62 becomes

$$\int_0^{R^*} (t - R^*)\phi(t)dt = \frac{R^{*2}}{2} + \alpha + \beta \log R^*. \tag{63}$$

Differentiating (63) twice with respect to R^*, we have

$$\phi(t) = -\left[\alpha + \frac{3\beta}{2} + \beta \log t\right]. \tag{64}$$

Hence from Eqs. 60 and 64 we have

$$V(\eta, R^*) = -\beta \log [(1-\eta)R^*] - \left(\alpha + \frac{3\beta}{2}\right). \tag{65}$$

Equation 65 is the logarithmic velocity distribution which does not however hold for $\eta = 1$. Usually one says that Eq. 65 holds only in the region $0 \leq \eta \leq 1 - \epsilon$, where ϵ is the thickness of a laminar sublayer. Since the logarithmic velocity distribution does not satisfy the boundary condition at the wall, it is not a rigorous solution of Reynolds equations.

IX. CONCLUSIONS

The Reynolds equations have been studied for the turbulent flows in circular pipe and the following conclusions may be drawn:

1. The correlation of the radial and the circumferential turbulent fluctuating velocity components $\overline{u'v'}$ is zero for the entire flow field.

2. The mean pressure distribution may be divided into two parts. One varies in a similar manner to that in laminar flow, that is, it decreases linearly with the axial distance along the pipe. The other part is influenced by the radial and the circumferential turbulent intensity

$\overline{u'^2}$ and $\overline{v'^2}$. From the experimental data on $\overline{u'^2}$ and $\overline{v'^2}$, we conclude that the mean pressure in the fluid is greater than or equal to that on the wall of the pipe.

From the equation for mean pressure, we see that near the center of the pipe, $\overline{u'^2}$ must be equal to $\overline{v'^2}$. It is verified by experimental results of Fage.

3. The mean velocity distribution and the correlation $\overline{u'w'}$ may be expressed by a polynomial of the radial distance, with the ratio of the shearing stress on the wall to that of the corresponding laminar shear stress of same maximum mean velocity as a parameter and an empirical constant. These expressions hold true all the way across the pipe, that is, both the turbulent region and viscous layer including laminar sublayer. The mean velocity distribution has been checked with Nikuradse's experimental data. The general shape of the theoretical curve agrees fairly well with the experimental data. But Nikuradse's experimental data near the wall are too high to be true, and the well-established condition of zero slip on the wall was not observed in Nikuradse's data, so that in order to check quantitatively the mean velocity distribution near the wall some refined measurements are needed.

4. The logarithmic mean velocity distribution is not a rigorous solution of Reynolds equations.

LIST OF SYMBOLS

a radius of the circular pipe
a_1 coefficient in mean velocity distribution (50)
A constant
B constant
C constant
C_f coefficient of friction defined by (26)
f any quantity of mean flow (2)
n exponential factor
p pressure
p_0 a reference pressure
P a component of pressure defined by (42)
r radial distance
$R_0 = U_0 a/\nu$ Reynolds number
$R^* = U^* a/\nu$ Reynolds number
$s = \tau_0/\tau_1$
t time
u radial component of velocity
u_i ith component of velocity
u_j jth component of velocity
U_m maximum velocity in the circular pipe
U_0 average mean velocity defined by (24)
$U^* = \sqrt{\tau_0/\rho}$ reference velocity
v circumferential component of velocity
$V = \bar{w}/U^*$ non-dimensional axial component of velocity
$V_m = \bar{w}/U_m$ non-dimensional axial component of velocity
w axial component of velocity

γ non-dimensional correlation coefficient defined by (29)
z axial distance
α constant
β constant
δ radial distance from the wall of the pipe
$\eta = r/a$ non-dimensional radial distance
θ angular displacement
ν coefficient of kinematic viscosity
$\xi = z/a$ non-dimensional axial distance
ρ density of the fluid
σ_1 non-dimensional radial turbulent intensity defined by (30)
σ_2 non-dimensional circumferential turbulent intensity defined by (31)
τ_l laminar shearing stress on the wall
τ_θ shearing stress on the wall
ϕ certain function
$\hat{\omega}$ non-dimensional pressure defined by (28)
— (bar) means average values
′ (prime) means fluctuating value
without bar or prime: instantaneous values.

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RADIO ASTRONOMY

BY

HANS HERBERT KLINGER[1]

INTRODUCTION

Since 1945 there has been much work done in the measurement and interpretation of cosmic or extra-terrestrial radio waves.[2] The first evidence of the existence of such waves was obtained twenty years ago by Jansky (1)[3] who observed, at short wavelengths, "random" noise signals whose intensity was a maximum when the antennas were directed at the center of the galaxy. In spite of this very remarkable astrophysical discovery, it was not until 1942 that scientists realized the great importance of cosmic noise, when it became evident that the sun is also a source of high-frequency radiation in the region of meter- and centimeter wavelengths. In recent years techniques of radio astronomy have been developed to enable investigators to obtain information about the positions of stellar bodies and matter in interstellar space that could never have been seen with an optical telescope. It is the aim of this paper to review the most important progress made in this new science of Radio Astronomy during recent years.

PRINCIPLES OF TECHNIQUES USED IN RADIO ASTRONOMY

To detect the very weak radio signals coming to the Earth from extra-terrestrial sources, a directive antenna and a highly sensitive radio receiver must be used. Figure 1 shows a typical receiving system.

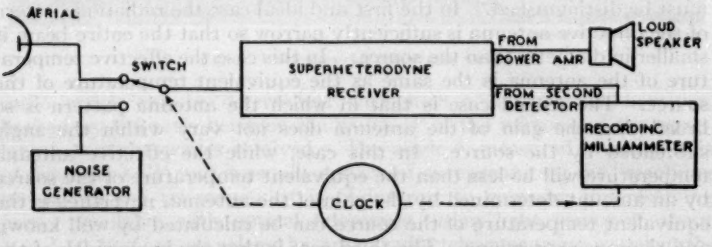

FIG. 1. A typical cosmic noise receiving system.

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[2] The term "cosmic radio waves" or "cosmic radio noise" includes solar radio noise, galactic and extra-galactic noise.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

Energy intercepted by the aerial is fed through a transmission line to the input of a superheterodyne receiver, where it is amplified linearly about a millionfold before rectification in the second detector, which is usually a diode. The d-c. output current actuates a recording milliammeter, which is calibrated in terms of input noise-power. At suitable intervals the receiver calibration is checked by being switched automatically to the noise generator, which is as a rule a diode operating under controlled temperature conditions. A clock provides electrical impulses both to the control mechanism of the switch and to the chronograph pen, providing charts of intensity *versus* time for various frequencies.

The basic physical quantity to be measured is the noise flux intensity S, defined as the total power per unit frequency interval falling on a unit area normal to the direction of the power flow. Using the M.K.S. system of units, it is expressed in watts per square meter per cycle per second. The noise flux intensity is related to the equivalent black-body temperature of the source by the relation

$$S = \frac{2kT}{\lambda^2}\Omega_s$$

which follows directly from the Rayleigh-Jeans approximation to Planck's radiation law for radio waves. In the equation above, T is the equivalent temperature (absolute) of the extra-terrestrial noise source, λ is the wavelength observed (meters), k is Boltzmann's Constant (1.37×10^{-23} Joule per degree) and Ω_s is the solid angle subtended by the source at the earth. For the case of the sun, $\Omega_s = 6.8 \times 10^{-5}$ steradian.

In measuring the intensity of the emission from extra-terrestrial sources, and hence their equivalent temperature, three different cases must be distinguished. In the first and ideal case the radiation pattern of the directive antenna is sufficiently narrow so that the entire beam is smaller in diameter than the source. In this case the effective temperature of the antenna is the same as the equivalent temperature of the source. The second case is that in which the antenna pattern is so broad that the gain of the antenna does not vary within the angle subtended by the source. In this case, while the effective antenna temperature will be less than the equivalent temperature of the source by an amount determined by the gain of the antenna, nevertheless the equivalent temperature of the source can be calculated by well known and rigorous expressions. The third case is that the beamwidth of the antenna lies between the two previous cases and is commensurate with the angular diameter of the source. Here, when the axis of the beam is directed at the center of the source, such as the sun, the gain varies across the disc of the sun, and therefore to compute the equivalent temperature of the sun from the measured intensity of the emission,

an integration involving a detailed knowledge of the pattern of the directive antenna is necessary.

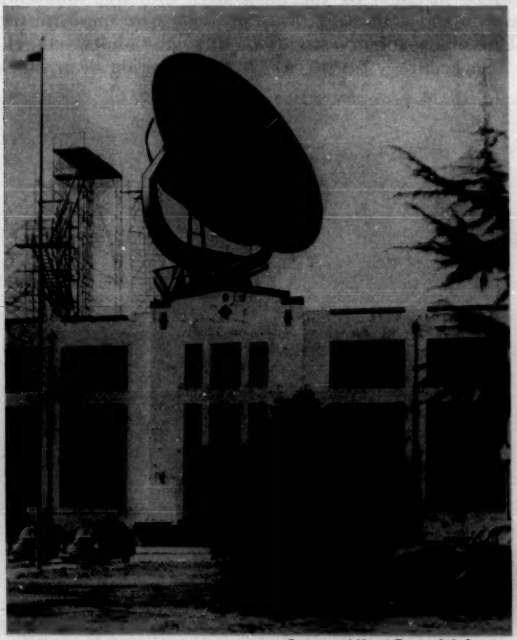
In Radio Astronomy it is common practice to use a parabolic mirror with a dipole antenna at the focus as a directive antenna. Such a "radio telescope," consisting of a parabolic wire mesh mirror, used at Cornell University for investigations of solar radio waves, is shown in Fig. 2. The accuracy θ with which the direction of arrival of cosmic

Courtesy of Cornell University

FIG. 2. Telescope for solar radio waves

radio waves can be determined with such a telescope is, however, limited by the ratio between the width W of the mirror and the wavelength λ of the waves received. The angle θ, measured in radians, is of the order λ/W, and is usually spoken of as representing the "resolving power" of the telescope. It is of interest to compare the resolving power available with optical telescopes, using light waves of about $\lambda = 5 \times 10^{-5}$ cm. wavelengths, and a radio telescope using wavelengths of about $\lambda = 5$ m. An optical telescope with an aperture of 10 cm. would have the same resolving power as a radio telescope with an aperture of 1000 km. It is obvious that, with radio telescopes, we cannot hope to approach the resolving power possible with light waves. In an attempt, however, to get the best possible resolving power, large mirrors

have been constructed. One of the largest radio telescopes used at present in Radio Astronomy is that of the Naval Research Laboratory, shown in Fig. 3, which consists of a parabolic mirror with an aperture



Courtesy of Naval Research Laboratory

FIG. 3. Giant telescope for cosmic radio waves.

of 60 ft. A much larger mirror, having an aperture of about 80 meters, is now in use at the University of Manchester. This telescope, when used on a wavelength of $\lambda = 2$ m., has a resolving power of about 1/40 radian, or 1.5°. Its importance in Radio Astronomy may be similar to that of the great 200-in. telescope used at Mt. Palomar in visual astronomy.

THE RADIOFREQUENCY RADIATION FROM THE QUIET SUN [4]

According to our present knowledge there are three types of solar radiofrequency radiation. First, there is a component which remains

[4] The term "quiet" sun means a sun which is free from sunspots, flares and other solar activities.

fairly constant in intensity and is randomly polarized. This component is due to thermal radiation or black-body radiation of the sun's atmosphere, which is a highly ionized hydrogen gas with a slight mixture of other elements—principally helium in the corona, and heavier elements in the chromosphere. In the Coulomb field of the positive protons, the free electrons approaching the protons are deflected and leave on hyperbolic orbits of lower energy. The difference in energy of these "free-free" transitions between adjacent orbits is transformed to electromagnetic radiation. The spectrum radiated is similar to the "Bremsstrahlung" of Röntgen-rays and consists of a continuum of wavelengths, whose longest wavelength region is that of radio waves. The intensity of this radiation however depends in a characteristic manner on the frequency. This is due to the fact that the emission and absorption of a black body, such as the corona, depends on the "optical depth"

$$\tau = \int_0^h k dh$$

which is the product of the absorption coefficient k and the length dh along the path transversed by the rays. The absorption coefficient of a proton-electron gas is a function of the electron density N, the wavelength λ, the temperature T and the refractive index n according to the relation

$$k = \frac{2\sqrt{2}\,\lambda^2 N^2 e^6}{3\sqrt{\pi}\,nc^3(mkT)^{\frac{1}{2}}} \log_e \left(\frac{4kT}{e^2\sqrt[3]{2N}} \right)^2,$$

where e and m are, respectively, the charge and the mass of the electron, k is Boltzmann's constant and c the velocity of light. From this equation, it is seen that the absorption coefficient of the corona is proportional to the square of the wavelength λ and the electron density N,

TABLE I.

Observer	Wavelength in cm.	Effective Temperature in degrees K.
Hagen	0.85	6,740
Dicke and Beringer	1.25	11,000
Southworth	3.2	16,000
Sander	3.2	22,000
McGready, Pawsey and Payne-Scott	10	25,000
Covington	10.6	58,000
McGready, Pawsey and Payne-Scott	25	150,000
Lehany and Yabsley	50	300,000
Reber	62.5	590,000
Pawsey	150	600,000
Ryle and Vonberg	172	ca. 1,000,000
Ryle and Vonberg	375	ca. 1,000,000
Machin	670	ca. 2,000,000

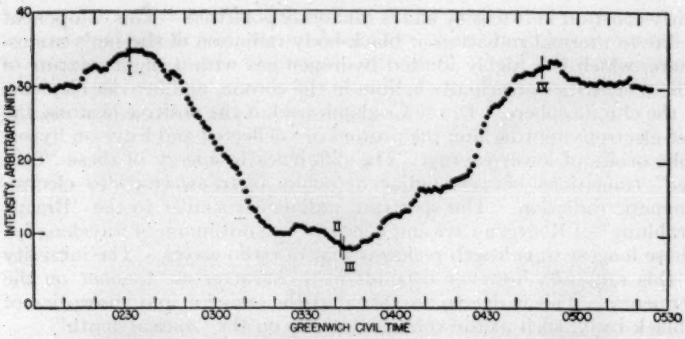


FIG. 4. Chart of intensity *versus* time for 65 cm. radiation received from the sun during its eclipse by the moon on September 11, 1950. Roman numerals indicate instants of first, second, third and fourth contact.

and inversely proportional to the 3/2 power of the temperature T. For light waves the corona is transparent. At meter wavelengths, however, the absorption of the corona—and hence according to Kirchhoff's black-body law also the emission—is such that a radio telescope receives approximately black-body radiation as determined by the electron temperature of the corona, a temperature on the order of a million degrees. For shorter radio waves the corona becomes more and more transparent. At 10-centimeter wavelengths radiation is partially emitted from the corona and the cooler chromosphere and at centimeter wavelengths or microwaves, radiation is emitted principally from the chromosphere, which acts as a black body for wavelengths of this magnitude. Hence the longer radio waves originate in the corona and the shorter radio waves in the chromosphere. In Table I the equivalent temperature of the quiet sun observed at various radio wavelengths is shown. From these measurements it is seen that, over the wavelength range observed, the equivalent temperature of the quiet sun varies from 7000 deg. Kelvin at the short wavelengths to the order of 10^6 deg. Kelvin at the longer wavelengths. These results are in accordance with the theories of black-body radiation of the sun developed by Martyn (2), Waldmeier (3) and Unsöld (4).

Because of the radio waves emitted by the sun's ionized atmosphere, the apparent diameter of the sun observed at radio frequencies is always appreciably greater than at visual wavelengths. Consequently, all solar eclipses viewed by a radio telescope appear to be annular rather than total. Results obtained from measurements at wavelengths of $\lambda = 3$, 10 and 65 cm. during the sun's eclipse by the moon (Fig. 4) indicate that the apparent diameter of the sun exceeds that of the moon

by 3, 7 and 11 per cent, respectively (5). At a wavelength of about $\lambda = 5$ m., the apparent diameter of the sun equals nearly twice the diameter of the visible sun.

ENHANCED SOLAR RADIATION

Besides thermal radiation from the quiet sun, during periods of sunspot activity there is an additional component, referred to as "enhanced radiation" associated with sunspots. This component is as a rule circularly polarized. There are, however, two different types of enhanced radiation associated with the wavelengths observed. At wavelengths below $\lambda = 50$ cm. the intensity of enhanced radiation appears to be closely related to the sunspot area, as shown for a wavelength of $\lambda = 10.7$ cm. in Fig. 5 (6). The intensity of this sunspot

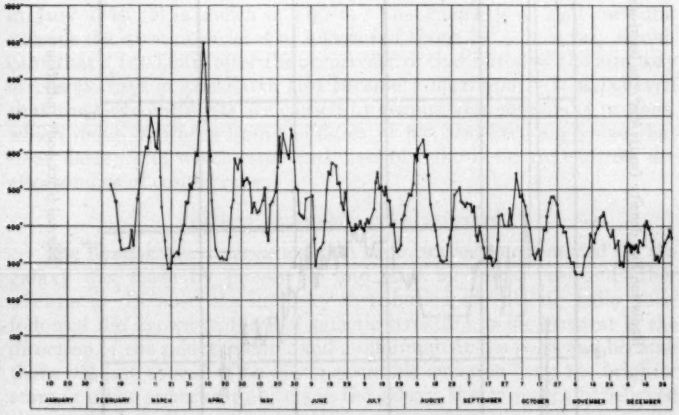


FIG. 5. The daily variation of solar intensity at 10.7 cm. wavelength for the year 1947. The antenna temperature is ordinate.

radiation exceeds the thermal component of the quiet sun only twice or three times. It is believed that this component originates in regions of coronal condensations, where the electron density and hence the optical depth is much greater than in the undisturbed corona of the quiet sun (7). As a consequence of this, regions of coronal condensations behave as a black-body for centimeter wavelengths. This hypothesis on the origin of enhanced radiation at microwaves seems to be more likely than those which consider it as radiation emitted by rotating electrons or protons in the magnetic fields associated with sunspots. The correlation between the intensity of enhanced radiation and sunspot area is observed down to shortest wavelengths of about $\lambda = 3$ cm. and

may be of considerable practical importance in conjunction with meteorological aspects.

At meter wavelengths another type of enhanced radiation is observed. This component is characterized by very rapid fluctuations of intensity. The intensity may be as much as 100 times greater than the basic thermal component at wavelengths of about $\lambda = 1$–2

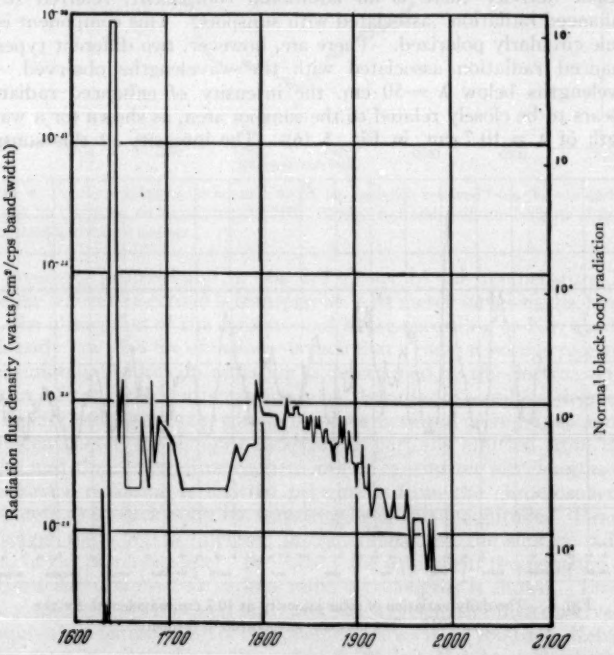


FIG. 6. Abnormal solar radiofrequency radiation (outburst) at $\lambda = 4.2$ m. wavelengths.

meters and 10^4 times greater at the longer wavelengths. This type of enhanced radiation has no correlation to the sunspot area; however, its intensity is usually a maximum when a sunspot is near central meridian passage and it changes the sign of circular polarization if the sunspot observed is passing through the central meridian (8). It is believed that this type of enhanced radiation is not due to thermal radiation as in the case of the short wavelengths. It seems to be much more likely that this component is due to a great number of discrete electrical bursts superposed on a basic thermal component.

BURSTS AND OUTBURSTS OF SOLAR NOISE

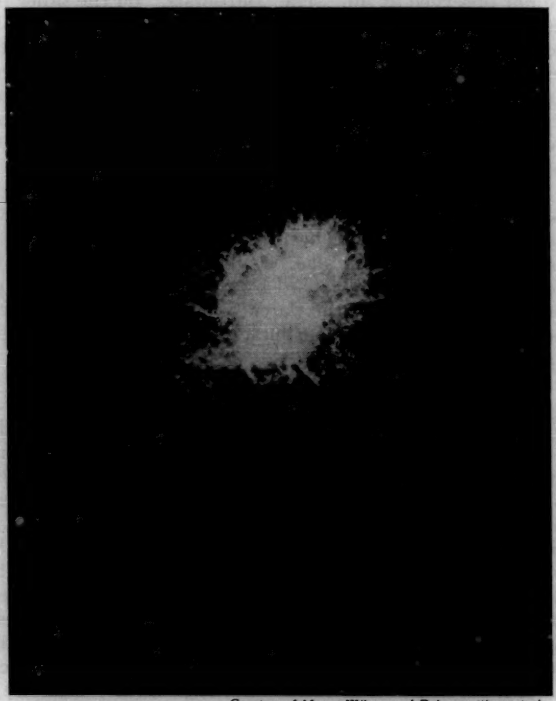
The third type of solar radio noise is characterized by sudden increases of intensity known as "bursts" and "outbursts." The bursts, whose duration does not exceed 30 seconds and may be as short as 2 or 3 seconds, occur at random. The intensity of the bursts decreases as the wavelength of observation decreases, so that in the microwave region they are not observed at all. The "outbursts," on the other hand are characterized by sudden increases of intensity to many times the quiet or background level, then fall exponentially, returning to the quiet noise level in from a few minutes to a few hours. The occurrence of an outburst is frequently coincident with the appearance of a solar flare and with a consequent sudden ionospheric disturbance on the earth, the so-called, "Mögell-Dellinger effect." A large solar outburst observed at a wavelength of $\lambda = 4.2$ m. during a period of great sunspot activity in July 1946 (9) is shown in Fig. 6. The intensity of that outburst exceeds the quiet noise level by a factor of about 10^8. It is very significant that a few hours after the occurrence of that outburst the intensity of cosmic rays at the Earth also became a maximum. It is believed that the large outbursts are caused by corpuscular streams of protons, which move outwards from the flares; it has also been suggested that solar flares, from which large outbursts of radio noise are emitted, are also sources of cosmic rays.

GALACTIC RADIO NOISE AND RADIO STARS

The remarkable observation that radio waves were emitted by the galaxy was made by Jansky (1) and later by Reber (10) and other scientists. Because the intensity distribution of galactic radio noise followed the known features of galactic structure, being greatest in the direction of the galactic center and a minimum at the poles and because there did not appear to be any measurable emission from the brighter stars or other prominent objects in the galaxy, it was generally believed that galactic radio noise originated in the interstellar gas due to the "free-free" transitions of electrons in the fields of protons. This theory was developed more fully by Henyey and Keenan (11). It was found however, that a reasonable fit with experimental observations could be obtained only for the shorter radio wavelengths ($\lambda < 1$ meter) but for the longer wavelengths the theory presented a serious discrepancy (12). It is only possible to explain the high intensities at the longer wavelengths on the theory of Henyey and Keenan by assuming an electron temperature of 100,000 to 200,000 deg. Kelvin, and this is quite in contrast to our knowledge of the electron temperature in the interstellar hydrogen gas concluded from visual observations. On the other hand, it was evident from the early measurements of Jansky and Reber that the intensity of galactic radio noise also could not be explained as being the aggregate black-body radiation from the stars in the galaxy. Even

after the discovery of the enhanced radiation from the sun, it was apparent that, if all the stars emitted enhanced radiation like the sun, then there still remained a factor of some 10^{10} to be accounted for in order to explain the intensity of the galactic radio noise.

A significant milestone in radioastronomy was the discovery that a considerable part of the galactic radio noise originates in discrete sources

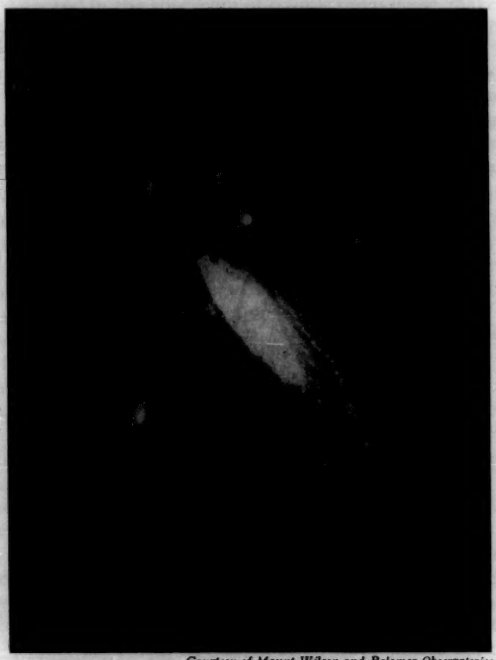


Courtesy of Mount Wilson and Palomar Observatories

FIG. 7. The Crab Nebula, taken with the 200-in. Hale telescope.

of radiation. At present there are about one hundred known discrete sources in the galaxy, from which radio waves are emitted. The remarkable feature of these "radio stars" is, however, that they do not coincide with visible stars or other prominent objects in the galaxy. Recent experimental evidence indicates that the whole of galactic radiation can be explained if the known radio stars are regarded as the most intense of a large number of radio stars, distributed throughout

the galaxy in a manner similar to the distribution of the visual stars (13, 14). We are thus faced with a possibility of the utmost cosmological importance, that the galaxy contains large numbers of stellar objects which are not visible and which only can be seen by radio telescopes.



Courtesy of Mount Wilson and Palomar Observatories

FIG. 8. The great nebula M 31 in Andromeda.

There have been several speculations as to the mechanism of these radio stars. Some astrophysicists believe that radio stars are young stars being not hot enough to emit light or that radio stars are stars which die. That radio waves are emitted from a dying star has been observed in the case of the Crab-nebula (Fig. 7). The Crab-nebula is a supernova, that is, a star whose explosion had been observed by Chinese astronomers in 1054. Today the Crab-nebula is a great cloud of hot gas expanding daily by 105 million kilometers. The very remarkable feature is, however, that the Crab-nebula is not only a source of intense radio noise but also a source of cosmic rays (15). Here, as

in the case of the large solar outbursts, there is a connection between cosmic radio noise and cosmic rays. It has been suggested that the discrete sources of radio noise in the galaxy might be responsible for the primary cosmic rays (16, 17, 18).

EXTRA-GALACTIC RADIO NOISE

The evidence for the existence of radio waves of extra-galactic origin from the great nebula in Andromeda (M. 31) (Fig. 8) which, at a distance of 750,000 light years, is the nearest of the extra-galactic nebulae, was obtained by Hanbury, Brown and Hazard (19). Although the intensity of the radiation received was only 1/150 of that from the intense radio star in the constellation Cygnus in the Milky Way, nevertheless, it is evident from these measurements that the mechanism

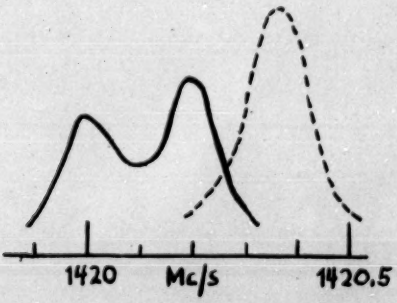


FIG. 9. Line profiles of the 1420 Mc/s emission line.

of the generation of extra-galactic radio noise is very similar to that of our own galaxy. In addition to the Andromeda nebula, several other extra-galactic nebulae have been observed to be sources of cosmic radio waves (20). It is estimated that the contribution of extra-galactic nebulae to the cosmic radio noise measured at the Earth is of the order of about 10 per cent.

Although at present the mechanism of cosmic radio noise is not understood, it is evident from recent research that cosmic radio waves are closely related to cosmic rays. Cosmic radio waves are the birth pains of cosmic corpuscular radiation. Therein lies the great importance of cosmic radio noise in astrophysics. It may be expected that the development of Radio Astronomy will help answer the important problem of the origin and genesis of cosmic radiation.

THE 21-CENTIMETER EMISSION LINE FROM INTERSTELLAR HYDROGEN

In addition to the study of radiation from the sun and from the radio stars, there is another important branch in radio astronomy which

is related to the study of interstellar matter by monochromatic radio waves emitted by its principal constituent, hydrogen. Such an emission line in the radio spectrum from interstellar hydrogen at 1420 Mc/s (λ = 21-cm) comes from a hyperfine-structure transition in the ground state of atomic hydrogen (21, 22, 23). The discovery of this line, the first to be detected in the spectrum of cosmic radio noise, makes possible for the first time in Radio Astronomy the study of Doppler shifts arising from relative motions of the source-region and the earth. The 1420 Mc/s line is found to be double in some directions (full line in Fig. 9). These directions extend over nearly a full quadrant of galactic longitude. The most interesting effect is a shift in frequency due to the differential rotation of the galaxy and the peculiar velocities of interstellar gas clouds. The radial velocity due to galactic rotation falls to zero towards the galactic center and in the other three cardinal directions. Hence in these directions the observed line-profile is a measure of the peculiar velocities in the line of sight. These velocities are found to be random and of the order of 15 km/sec. In any other direction in the galactic plane, the frequency of the radiation is affected by galactic rotation, and depends on the distance to the source-region. Thus, if the effects of galactic rotation and random motions can be separated, observed frequency shifts can be interpreted as distances.

The observed characteristics of the double line are most simply explained as an effect of galactic rotation. In this case, the source must form two vast elongated masses of hydrogen, possibly delineating two spiral arms of the galaxy. Previously, by far the greater part of interstellar matter was undetectable by any known means. This new development offers a striking example of the important role of radio studies in astronomy.

Acknowledgments

The author wishes to express his sincere gratitude to Prof. C. R. Burrows of Cornell University and to Dr. J. P. Hagen of the Naval Research Laboratories for the photographs of the Radio Telescopes used in this paper.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

LIGHTNING RESISTANCE OF PORCELAIN INSULATORS

Integrated with the electrical standards program of the National Bureau of Standards is an investigation of techniques for determining the performance of electrical equipment under extreme conditions. One phase of the investigation, dealing with the action of pin-type porcelain distribution insulators during high-intensity electrical storms, has resulted in the development of a method for measuring the ability of these insulators to withstand lightning strokes. The method involves the generation of voltages similar in waveform to those appearing on lightning-struck power lines and the discharge of these potentials across an insulator at rates up to 11 million volts per microsecond. The study, sponsored by the Rural Electrification Administration, was conducted by J. H. Park and H. N. Cones of the NBS staff.

In some rural areas it is not economically feasible to protect the high-voltage distribution lines against direct strokes of lightning. In most cases a lightning stroke flashes over the insulators, and after the momentary ground has been cleared, the line operates normally. Occasionally, however, an insulator is punctured during a lightning storm and will shatter. In other instances, after being struck by lightning, the insulator has all outward appearances of intactness but actually has been punctured through the porcelain from a point under the tie wire or line conductor to the supporting pin. During a dry period the insulator performs its designated function; but during a rain storm the water creates an electrical path over the surface of the wooden crossarm on which the insulator is mounted, and the punctured insulator is unable to prevent a ground.

Field experience has indicated that most of the failures occur on certain insulator designs that have previously performed favorably under the standard laboratory investigating methods currently in use. One insulator manufacturer,[1] however, has been using a measurement technique which consists in subjecting the insulator to a very steeply rising voltage surge (rates of rise from 3,000 to 10,000 kv. per microsecond). When exposed to the voltage surges, the insulators that will not withstand lightning strokes puncture and shatter. Because the methods for producing and measuring such steep-front voltage surges have not been standardized, the National Bureau of Standards has been requested to

* Communicated by the Director.

[1] The Ohio Brass Company independently employs techniques similar to those designed by NBS to evaluate the performance of its insulator products. Comparable results have also been obtained.

make an investigation that would ultimately lead to the adoption of some standard technique.

The experimental program conducted by the National Bureau of Standards was carried out in two phases: (1) a suitable measurements method or procedure was designed, and (2) the method was applied to various types of new insulators so that a comparison between the NBS procedure and other existing techniques could be made. The voltages required to simulate the effect of lighting strokes were obtained from a surge generator consisting of twenty $\frac{1}{3}$-microfarad capacitor units, each rated at 100 kv. The major objective was to obtain a voltage wave rising abruptly from zero at a very high and constant rate until insulator flashover occurred. The arrangement chosen was a parallel circuit with 10 series-connected capacitors forming each arm. With this technique it is possible to obtain a high rate of voltage rise and maintain a maximum voltage output of 1,000 kv. by reducing the effective internal impedance of the generator. A 10,000-ohm resistor connected to ground and in parallel with the capacitor bank serves to keep the external discharge circuit at a fairly low voltage until the current at the gaps of the capacitors has increased to several amperes. An external resistance of 70 ohms is connected between the top of the generator (20 feet above the floor) and an output discharge gap. The other side of the gap joins a voltage divider, used to measure the applied potential, and the insulator to be investigated. When the current in the capacitor bank rises to a sufficiently high value, the air in the gap breaks down and the steeply rising voltage is applied across the insulator and the divider. The average rate of voltage rise on the front of these waves is 7,000 kv. per microsecond, but by shorting out the internal resistance of the generator and by removing the external resistance, rates of voltage rise up to 11,000 kv. per microsecond have been obtained.

During the preliminary investigations, no punctures were obtained on the first application of a surge voltage to an insulator, even when the steepest rates of voltage rise were used. However, several insulators punctured after repeated applications, some with rates of voltage rise as low as 3,000 kv. per microsecond. Consequently, in the method finally selected eight equal voltage surges are applied at 20-second intervals to each insulator.

Further studies of measurement techniques and procedures utilizing steep-front voltage waves are now in progress at the National Bureau of Standards. Before standards can be adopted by the industry, however, the various laboratories of the country must agree on experimental procedures for generating and measuring the necessary voltage surges.

RECLAMATION OF ASBESTOS

Simple, effective procedures for reclaiming asbestos from discarded pipe insulation have been developed by E. W. Zimmerman of the

National Bureau of Standards. Extraneous material is removed from the asbestos cloth by chemical treatment, and the cleaned cloth is reduced to fiber form in a paper-pulp beater. The recovered asbestos is unchanged chemically, with little or no shortening of the fibers. It appears well suited to further use as electrical and heat insulation, for making asbestos paper, and as a filler in molded plastic compounds.

Because of its incombustibility, strength, and low conductivity, asbestos is widely used in roofing shingles, fireproof paints, stove and building insulation, heat-resistant aprons and gloves, and brake linings. However, present supplies of this raw material are critically short. As large quantities of molded asbestos and asbestos cloth are used to insulate pipes of various kinds in naval vessels, the Navy Bureau of Ships requested NBS to study the possibilities of reclaiming asbestos insulation discarded during the repair or refitting of ships.

To aid the Bureau in its investigation, the Navy supplied samples of discarded pipe insulation. These were of three general types: asbestos cloth, asbestos-cotton cloth, and a molded pipe lagging having asbestos fiber as a filler and magnesia cement as a binder. Some of the asbestos-cotton cloth had glass strands woven into the yarns. Other samples of this cloth were painted or were covered with a cement-like material containing magnesia. The problem thus reduced itself to two parts: first, removing the extraneous material and, second, reducing the cleaned yarns or cloth to fibrous form.

For applications where flexibility is desired, such as asbestos paper or cloth, chrysotile asbestos is used. Although this type of asbestos is quite satisfactory for insulating heated pipes, it loses its water of constitution and becomes brittle when heated to about 800° C. It was therefore necessary to develop procedures that would reclaim the asbestos fibers without the use of excessively high temperatures.

It was found that asbestos or asbestos-cotton fabrics other than those containing paint can be freed of adhering cement by first treating with 5-per cent hydrochloric acid and then rinsing. Fabrics painted on one side are treated with alkali instead of acid to remove the paint as well as the cement. This is accomplished by boiling in 5-per cent sodium hydroxide solution for 15 to 30 minutes, followed by a preliminary rinsing, a warm wash with a synthetic detergent to remove the paint pigments, and a final thorough rinsing.

Removal of cotton from the cleaned cloth is best effected by the use of heat, but the process requires rather careful control to avoid loss of the water of constitution and consequent embrittling. Thin layers of the material are heated in a muffle furnace at a temperature between 400° and 450° C. The time necessary to burn out the cotton in this temperature range depends largely on the equipment used. After removal of cement, paint, and dirt, the cloth is reduced to fiber in a rotary food blender or in a paper pulp beater.

The fabric containing glass strands was composed of very short asbestos fibers. While an acid wash followed by rinsing removes most of the cement from this material, removal of the dirt is difficult without loss of some of the short asbestos fibers, which float off with the dirt. The recovered product contains glass fibers intermixed with the short asbestos fibers and thus will probably be of limited use.

The molded insulation yielded only 15 per cent of fiber, and this was highly contaminated with foreign materials. Thus, recovery of asbestos from such material does not seem very practicable. In any case, use of the fibers recovered from the molded insulation would have to be limited to magnesia-cement components and similar products.

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JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

BROOKS, H. B.: Electrical Indicating Instruments Used in Early Edison Central Stations.
BLASCHKE, ALFRED C. AND CRAIG L. TAYLOR: The Mechanical Design of Muscle-Operated Arm Protheses.
MOON, PARRY AND DOMINA EBERLE SPENCER: The Coulomb Force and the Ampere Force.
SKILLING, HUGH H. AND JOHN C. BECKETT: Control of Air Ion Density in Rooms.
SEAL, PHILIP M.: Analysis of Network Response to Periodic Laws.
TOMOVICH, RAJKO: A Versatile Electronic Function Generator.
BULAND, R. N.: Analysis of Nonlinear Servos by Phase-Plane-Delta Method.

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NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, EXECUTIVE DIRECTOR

JAMES WATT (1736–1819)

It is difficult to say how much power is produced daily in the United States but it is fairly safe to say that it is less than one hundredth the cost of the equivalent manpower, even when man is given the minimum wage. The story of the development of mechanical power is fascinating, and visitors to the Museum have especial opportunity for tracing the story through its major steps.

In this age of the internal combustion engine we are apt to overlook the significance of the part played by the steam engine, which brought about the Industrial Revolution and moulded our current civilization. No invention has exceeded in importance James Watt's steam engine. Nothing has surpassed its influence for making man's physical efforts count for less and less.

Watt did not invent the steam engine. As in the case of other great inventors he built upon the successes and failures of his predecessors. The humble Scottish instrument maker had the vision to perceive that Newcomen's slow, noisy, and uneconomical atmospheric engine could be improved to the point where it could render obsolete all the windmills and water wheels driving the machinery then in use.

His first inspired improvement was to redesign its construction so that it could be operated more economically and smoothly. The defect in Newcomen's engine resided in the alternate heating and cooling of the cylinder. This was remedied by Watt when he provided a separate cylinder for the condensation of the steam. He kept his cylinder always hot. This was probably the greatest single achievement in the history of steam engineering. The efficiency of the engine was immensely improved.

But that was not Watt's sole contribution to the development of the steam engine. Newcomen had only been able to achieve a direct reciprocal motion, up and down, which made his engine useful for operating a pump. It had no other application. Watt made his engine capable of rotary motion and thereby made it applicable to innumerable operations. Again it must be mentioned that Watt was not inventing, but adapting. He did not invent rotary motion. It is a tenable supposition that his wife used a spinning wheel, but if this was not a part of the domestic furnishings, he must have been familiar with windmills and water wheels. He made the steam engine capable of rotary motion by means of toothed wheels in his "Sun and Planet" engine which the Museum visitor can examine in an excellent reproduction.

The next improvement introduced by this ingenious Scot was of equal importance. Hitherto, the steam engine had been operated by

the creation of a vacuum under the piston, which then fell in the cylinder through atmospheric pressure. Watt made use of the expansive power of steam by applying it to each face of the piston alternately, thereby greatly increasing the power of his engine.

Watt left other impressions of his genius upon the science of steam engineering. He introduced the term "horse power" into technical language and furnished the definition which is used to this day. He also introduced the fly-ball governor for regulating the speed of engine

Watt's "Sun and Planet" engine, first brought to this country by the Manhattan Company for the New York City waterworks in 1799 or 1800.

operation, and the steam indicator which enables the engineer to learn what is going on inside the cylinder.

These were some of Watt's direct contributions to engineering. The indirect improvements encouraged by his developments are immeasurable. It was the demand for steam engines and the machinery they operated which led to the creation and development of the machine tool industry; it was the steam engine that promoted a hastening of travel by land and water; it was the steam engine that eased man's burden of physical labor. Watt's "Sun and Planet" engine deserves your attention and reverence next time you visit the Museum.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

DR. NICOL H. SMITH, DIRECTOR

EXPERIMENTAL IRON CARBONYL POWDERS*

BY

FRANK E. JAUMOT, JR. AND LEONARD MULDAWER

I. Strain and Crystallite Size in the Untreated Powders

Researchers in the Solid State Physics Division have determined the crystallite size and strain present in an experimental iron powder[1] produced by a carbonyl process. Electron microscope examination of the powders indicated a mean particle diameter of about 20,000 Å.

The method used to determine the strain and crystallite size has been described by Warren and Averbach (**1, 2, 3, 4**). Using a combination of Fe K_α and Mo K_α radiation, the shapes of nine powder diffraction lines were obtained for the powder and also for a well-annealed "standard." From a Fourier analysis of these lines (corrected for instrumental broadening) obtained with the aid of Lipson-Beevers strips, the coefficients of line profiles due only to strain and crystallite size broadening were obtained. A curve of the logarithm of these Fourier coefficients as a function of the order of the diffraction lines gave the crystallite size and strain directly.

The analysis showed that this material is anisotropic both in crystallite dimensions and strain. For the (100) direction in the crystal a mean size of 50 Å was obtained; for the (110) direction, 70 Å; and for the (111) direction, 80 Å. The strains measured in these directions were approximately 3.5×10^{-3}, 6.3×10^{-3}, 2.2×10^{-3}, respectively. If we can assume a Hooke's law, a very uncertain assumption, the stress in the (110) direction (these are the slip planes in iron) corresponding to the strain, is of the order of 100 tons/in.^{2}, or roughly the yield stress in iron.

II. Annealing Kinetics

In order to study the annealing characteristics of the above material, powders were annealed for two hours at temperatures of 150, 250, 300, 350, 400 and 450° C. Line profiles were obtained and analyzed to determine the particle size and strain. In addition the relative peak heights, half-breadths, and line positions were determined as functions

* Supported in part by the Office of Naval Research. This is an abstract of a paper that has been submitted for publication to *Acta Metallurgica*. Part I was given at the North Carolina Meeting of the American Physical Society, March 1953; and Part II will be given at the Pittsburgh Diffraction Conference in November 1953.

[1] General Aniline and Film Corp., designated P818.

of annealing temperatures. It was found that annealing produced a diminution in the lattice parameter accompanied by a corresponding reduction in the strain. The recovery is complete within experimental error, at about 400° C. Crystallite size increased monotonically from 70 Å in the untreated powder to over 1200 Å in the powder annealed at 450° C. The major portion of the crystal growth occurs at temperatures above 350° C, which, however, are considerably below the normal recrystallization temperatures of iron-carbon alloys.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

A COLORIMETRIC METHOD FOR ESTIMATING 2-DESOXYGLUCOSE

BY

FRANCIS B. CRAMER AND GEORGE A. NEVILLE

In connection with studies of the physiological effects of 2-desoxy-D-glucose (2DG), a method of analyzing vital fluids for their 2DG content was required. The first method tried, involving periodate oxidation and development of a colored ferric chloride complex with the resulting malonaldehyde, had to be abandoned because of interference from non-precipitable constituents in rat urine.

The procedure now in use is based on a quinaldine reaction for alpha-methylene aldehydes, first used analytically by Velluz, Amiard and Pesez (1). The color produced by 0.01 to 0.2 mg. of 2DG is easily read in a Klett-Summerson colorimeter. The precision of the method in this sample range is about ± 10 per cent (Fig. 1). It is possible that

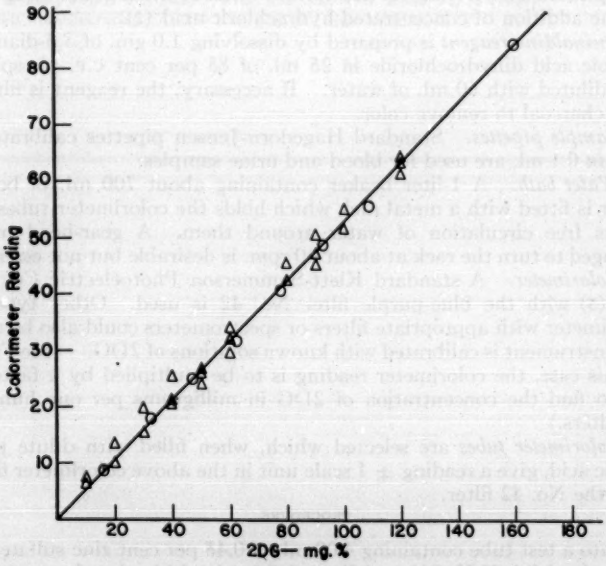

FIG. 1. Analyses of solutions containing known amounts of 2DG dissolved in water, ○, and in oxalated rat blood, △.

fluorometric methods would yield greater precision (2), but the required equipment is not generally available. The 2-desoxy pentoses do not give a color, presumably because of their extreme acid sensitivity. Consequently desoxyribonucleic acid (DNA) and its break-down products do not interfere. Relatively large amounts of glucose (up to 5 mg.) and other substances in deproteinized blood do not interfere. Urine should preferably be diluted 10:1 before analysis. Rat urine seriously contaminated with food and feces can not be analyzed quantitatively. It is good practice to base all colorimeter readings on a blank made with a similar sample known to contain no 2DG.

MATERIALS AND APPARATUS

Deproteinizing reagents: 0.1 *N* sodium hydroxide and 0.45 per cent zinc sulfate are prepared as in the Hagedorn-Jensen method for blood sugar (3).

3,5-Diaminobenzoic acid dihydrochloride is prepared by the catalytic hydrogenation of 3,5-dinitrobenzoic acid suspended in absolute alcohol (4), with Raney nickel as catalyst. For purification, the material is precipitated from a charcoal-decolorized water solution of the compound by the addition of concentrated hydrochloric acid (2).

Quinaldine reagent is prepared by dissolving 1.0 gm. of 3,5-diaminobenzoic acid dihydrochloride in 25 ml. of 85 per cent C.P. phosphoric acid diluted with 60 ml. of water. If necessary, the reagent is filtered with charcoal to remove color.

Sample pipettes. Standard Hagedorn-Jensen pipettes calibrated *to contain* 0.1 ml. are used for blood and urine samples.

Water bath. A 1-liter beaker containing about 700 ml. of boiling water is fitted with a metal rack which holds the colorimeter tubes and allows free circulation of water around them. A gear-head motor arranged to turn the rack at about 60 rpm. is desirable but not essential.

Colorimeter. A standard Klett-Summerson Photoelectric Colorimeter (5) with the blue-purple filter No. 42 is used. Other types of colorimeter with appropriate filters or spectrometers could also be used. The instrument is calibrated with known solutions of 2DG. (See Fig. 1. In this case, the colorimeter reading is to be multiplied by a factor of 1.9 to find the concentration of 2DG in milligrams per one hundred milliliters.)

Colorimeter tubes are selected which, when filled with dilute phosphoric acid, give a reading ± 1 scale unit in the above colorimeter fitted with the No. 42 filter.

PROCEDURE

Into a test tube containing 4.00 ml. of 0.45 per cent zinc sulfate and 0.80 ml. of 0.1 *N* NaOH is delivered 0.1 ml. of blood, urine, or other fluid containing 0.01–0.15 mg. 2DG. The pipette is rinsed several times

with the mixture in the test tube. The test tube is placed in boiling water for 3 min., then cooled in a cold water bath. The contents are then filtered through a *dry* paper in a dry funnel, into a dry test tube; 3.00 ml. of the filtrate are added to 3.00 ml. of the quinaldine reagent in a dry colorimeter tube. The tubes are stoppered loosely with corks and heated in boiling water for exactly 15 min., after which they are cooled in cold water for 15 min., wiped dry and inverted to mix the contents. The colorimeter is adjusted to zero on a blank which has been run exactly the same way as the tests, but with a sample known to contain no 2DG. The amount of color developed in the tubes is then read and the reading multiplied by the proper factor to obtain the amount of 2DG in the original sample.

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BOOK REVIEWS

PRINCIPLES OF MATHEMATICAL ANALYSIS, by Walter Rudin. 227 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $5.00.

The turning point in the education of mathematicians is the transition from the undergraduate level to the stage at which rigor and conceptual understanding predominate. Although a sample of watered-down epsilon-delta methods is usually handed out during the calculus, it is the prerogative of the advanced graduate student to think exclusively in terms of rigor. To this end, Professor Rudin has written his *Principles of Mathematical Analysis* for the International Series in Pure and Applied Mathematics. It is the author's first major venture into the textbook field, but it promises to be a success. Designed to serve the pure mathematicians, this book will also be a treat for the applied mathematician who can profit from such an experience. The topics for the ten self-contained chapters are selected judiciously from the vast field of advanced analysis of real and complex variables. The style is lucid, yet terse and powerful. The reader should forever be on guard not to miss a point, nor to overlook the full import of a concept. Repetition is only an undergraduate didactic tool. This book's emphasis is on clarity of concepts with a minimum of recapitulation.

The first chapter introduces the real and complex number systems. The use of notations, definitions, and theorems is well ordered here and elsewhere in the book. It creates a favorable impression from the very outset. The next chapter treats elements of set theory and concludes with the Heine-Borel Theorem and the Cantor Set. The third chapter deals with numerical sequences and series which lead to the Bolzano-Weierstrass Theorem and monotonic functions in the fourth chapter on continuity. In the fifth chapter on differentiation are found the mean value theorems, L'Hôpital's Rule, and Taylor's Theorem.

Next to the last, the sixth, chapter on Riemann-Stieltjes integration is possibly the most elegant. The notion of the partition of intervals is followed by theorems for bounded functions over partitioned intervals from which the Riemann and Stieltjes Integrals are developed. The introduction of the unit step function, finally, permits the author to observe for the benefit of applied mathematicians that the moments of inertia for continuous and discontinuous mass distributions are both representable as Stieltjes Integrals. The seventh chapter extends earlier results to sequences and series of functions. Further work leads to the Theorem of Weierstrass and the generalizations of Stone for algebras of continuous functions on compact sets. Noteworthy is example 7.4 of an everywhere discontinuous limit function which is not Riemann integrable.

The eighth chapter deals with further topics in the theory of series, namely with the exponential, logarithmic, and trigonometric functions and with more advanced aspects of Fourier series and orthonormal systems. The ninth chapter on functions of several variables studies first metric spaces and their properties, then continuous functions of several real variables, vector-valued functions, inverse and implicit functions, and functional dependence. The last chapter is the longest and also the strongest. It outlines the fundamental concepts of the Lebesgue theory of measure and integration which are especially important to the student of probability theory. Here is the background he needs for the analysis of stochastic processes. The chapter concludes with the theorems of Fatou and Riesz-Fischer.

The whole volume is keyed to abstract mathematics. The author gives the reader only an occasional hint how to visualize certain concepts. It might have been helpful had he included more such hints. Examples and exercises are well chosen and support the development of a sound theoretical structure, especially where they are in form of counter-examples. As this is a graduate text, no "answers" are given to the exercises. Minor errors were noted on page 65 where the third line should read "for" instead of "or," and on page 106 where the line following equation (55) should refer to (55) instead of (51). The

complete lack of diagrams may be mourned by those who prefer to think in terms of concrete rather than abstract models. But too many *pontes asinorum* might have spoiled the classical flavor of the book.

Professor Rudin's text is written in keeping with the traditionally high standards of the International Series. There seems to exist a comfortable market for class room adoptions, but the volume appears also suitable for home study by the graduate student. In addition, it will be of value to research personnel in fields that require a ready reference to the many theorems of advanced mathematical analysis.

CARL HAMMER

ALUMINUM IN IRON AND STEEL, by Samuel L. Case and Kent Van Horn. 478 pages, illustrations, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $8.50.

Many non-ferrous metals are added to iron to impart favorable properties to the resulting steel. In some cases these additions are made to increase the strength parameters, increase the corrosion resistance, etc. In others the additions are made in order to assist in the removal of harmful elements such as oxygen. This book is the first in a new monograph series designed to describe in detail the role that certain non-ferrous additions play in the science of iron and steel making.

Although aluminum is of major importance in modern ferrous metallurgy it is interesting to note that less than twenty years ago aluminum additions were regarded only as a means of correcting for or concealing poor furnace practice by quieting overoxidized melts. The authors have reviewed the field and have critically selected information that shows why aluminum is no longer considered in such an unfavorable light.

The book is divided into two parts. The first part describes aluminum as a deoxidizer in iron and metal, and the second part describes the effect of aluminum as an alloying addition.

Part I contains a particularly good description of the inclusions in steels deoxidized with aluminum. Austenitic grain-size control is discussed in detail. Such control has long been recognized to be of great importance in the resulting properties of the iron or steel. However, it is unfortunate that the authors did not discuss the effect of aluminum on ferrite grain size in irons and low carbon steels. Austenitic grain size has little meaning for such material as compared to ferrite grain size since ferrite is the stable phase under service conditions. Notch sensitivity and ageing in steel are discussed both in general terms and specifically as affected by aluminum. These sections serve to increase the importance of the book as a reference source.

Part II presents in detail important data on the alloying effect of aluminum in steel. Here the authors discuss such subjects as the constitution of iron-aluminum alloys, heat resisting and permanent magnet aluminum alloys and aluminum coatings on steels.

The book has an excellent index, table of contents, and is appended by a list of 341 references. It sets a high standard for the succeeding volumes and should prove to be an invaluable aid to those interested in steel making.

R. L. SMITH

MAN IN SPACE, by Heinz Haber. 291 pages, drawings, 15 × 22 cm. New York. Bobbs-Merrill Co., Inc., 1953. Price, $3.75.

Space travel is a new and fascinating subject. Only in the last two decades has it attained the stature of a science. Prior to that time it was looked down on as science fiction. In this new development in which man will strive to conquer and explore space, much has been written of the technical achievements which are necessary before this becomes a reality. However, today, with airplanes which fly at altitudes of 80,000 feet and speeds of 1,300 miles per hour, the scientist and designer of these vehicles are suddenly aware that the weakest link in the entire picture is earth-conditioned man.

Take man from the earth and immediately he is plunged into an environment which is completely alien to him. And, unless special precautions are taken and special devices prepared, he can never survive this ordeal.

To explore man's position in this new environment there has appeared one of the most significant books of recent times. Written by Dr. Heinz Haber, of the Department of Engineering at the University of California, *Man in Space* is an analysis of the physical and

psychological hazards which man faces in leaving the earth. Because there is no laboratory available to test reactions which will be encountered in space, most of the work being done in this field is an extrapolation of what we know about man and his environment on the earth. As a result, much of what is written is speculative and to the credit of the author he firmly and definitely indicates this in his writings.

But speculation must play an important role in this analysis. Only in that fashion can there be progress. Dr. Haber quotes from Thomas Henry Huxley to substantiate this reasoning. Huxley wrote, "Those who refuse to go beyond fact rarely get as far as fact . . . Almost every great step in the history of science has been made by the 'anticipation of nature,' that is by invention of hypotheses which, although verifiable, often had very little foundation to start with."

Only by straying from experimental fact can the picture of the future be painted though it should be pointed out that the foundation of the speculation is based on our tremendous wealth of knowledge of physics, medicine and engineering.

Man in Space is a well written book which merits and will probably receive wide circulation. It is the only book which has listed in it the complete problem which man will encounter in space and the chances for the survival under these hazardous conditions.

I. M. LEVITT

GENERAL BIOCHEMISTRY, by William H. Peterson and Frank M. Strong. 469 pages, tables, illustrations, 16 × 24 cm. New York, Prentice-Hall, Inc., 1953. Price, $6.50.

General Biochemistry presents a balanced introduction to the chemical activities of plants, animals, and microorganisms. This book is intended as a text for students with a knowledge of general and organic chemistry. With the exception of the chapters on biological energetics and acidity, it is essentially nonmathematical and does not require a background of physical chemistry. It emphasizes the facts on biochemical compounds and their interrelationships in living systems. The methods by means of which these facts were discovered and verified are briefly mentioned in the introductory chapter.

The first part deals with the materials of living organisms from the standpoint of descriptive chemistry. The first constituent discussed is water, the most abundant but by no means inert substance of protoplasm. The classifications and properties of carbohydrates, lipids, proteins, and nucleic acids are presented in a conventional manner. The properties and functions of minerals, vitamins, and hormones are clearly described with due consideration to recent advances as well as to long established facts. The chapter on enzymes, which was written by G. W. E. Plaut, includes a carefully prepared abridged classification of enzymes and an up-to-date condensed description of some of their characteristics. More attention could have been given, however, to the kinetics of enzymatic activities, which, in the reviewer's opinion, is the logical foundation for the dynamic aspects of metabolism.

The second part considers the functions, interconversions, and interactions of the chemical constituents of living cells. The discussion of metabolism is subdivided according to the type of organism: animal, plant, and microorganisms. Because of the many differences in metabolism among these three groups, this subdivision is desirable. However, some of the metabolic patterns, like their chemical components, are ubiquitous. The similarities and the differences are skillfully presented. Some of the major subjects in the chapter on animal metabolism are glycolysis, citric acid cycle, cytochrome system, oxidation of fats, and metabolism of amino acids. The growth requirements, intermediary metabolism, and end products of representative bacteria, yeasts, and molds are comprehensively discussed. The production of antibiotics and other useful products by microorganisms is briefly considered.

Worthy features are the 46 compact informative tables, which present data on economic importance of products from living things, properties of compounds, chemical composition and energy value of food, and other pertinent topics. Each chapter has a list of review questions and a list of references and suggested reading.

W. D. YUSHOK

FLUID DYNAMICS, Vol. IV, edited by M. H. Martin. 186 pages, diagrams, 18 × 26 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $7.00.

This volume contains fourteen papers presented at the Fourth Symposium on Applied Mathematics held at the University of Maryland June 22–23, 1951, and cosponsored by the U. S. Naval Ordnance Laboratory, White Oak, Maryland. The subject of the Symposium was Fluid Dynamics, and the four sessions were devoted to Turbulence, Compressible Flow, Foundations, and Incompressible Flow. The publication in book form of these recent contributions by recognized authorities should provide a source of much stimulation for workers in this field.

"Some Aspects of the Statistical Theory of Turbulence," by S. Chandrasekhar, contains a discussion of isotropic tensors and the equations of isotropic turbulence, axisymmetric turbulence, magneto-hydrodynamics, and fluctuations of density. It concludes with a derivation showing that eddies in the density fluctuations which are larger than a certain critical size will be amplified, thus showing a tendency toward disintegration and formation of condensation in the medium.

"A Critical Discussion of Similarity Concepts in Isotropic Turbulence," by C. C. Lin, is concerned with two closely related concepts of similarity of the spectrum; the one asserts that the spectrum is completely similar during the process of decay, and the other asserts that there may be lack of similarity at the lower end of the spectrum.

"The Nonexistence of Transonic Potential Flow," by Adolf Busemann, shows that this property is not connected with the nonlinear differential equation in the physical plane or with the difficulties in determining the physical boundary conditions if the hodograph method is used. The nonexistence of a regular solution is actually a property of the linear differential equation of the mixed elliptical and hyperbolic type and the required boundary conditions.

"On Waves of Finite Amplitude in Ducts," by R. E. Meyer, is an abstract (4 pages) of a theory published, in two parts, in *Quart. J. Mech. Appl. Math.*, Vol. IV (1952), pp. 257–291.

"On the Problem of Separation of Supersonic Flow from Curved Profiles," by T. Y. Thomas, contains both theoretical and experimental results and indicates where further experimental work is needed.

"On the Construction of High-Speed Flows," by G. F. Carrier and K. T. Yen, contains a new method of construction of flow patterns in which sonic or nearly sonic speeds characterize a portion of the flow field. It has the advantage that the flow is represented in closed form in the hodograph variables.

"An Example of Transonic Flow for the Tricomi Gas," by M. H. Martin and W. R. Thickstun, contains a study of the equation of state of a compressible fluid whose flow is governed by Tricomi's equation, obtaining, among other results, a theorem about equations of state, first obtained, by another method, by S. C. Wang.

"On Gravity Waves," by Albert E. Heins, is concerned with the boundary-value problem for waves in channels of finite or infinite depth with certain special obstacles which make possible the explicit solution of the problem.

"Hydrodynamics and Thermodynamics," by S. R. De Groot, presents a development of the hydrodynamical equations from the viewpoint of the thermodynamics of irreversible processes. This procedure allows an insight into the fundamental concepts and the logical structure of hydrodynamics.

"Nonuniform Propagation of Plane Shock Waves," by J. M. Burgers, contains short discussions of a few cases of nonuniform propagation of plane shock waves. The Lagrangian description of the motion is employed in which the path of an element of fluid is studied rather than the velocity at a coordinate position.

"Theory of Propellers," by Theodore Theodorsen, contains a brief exposition of the propeller theory, giving exact expressions for thrust torque and efficiency based on the knowledge of the flow function φ, as it appears far behind the propeller. The expressions obtained depend almost entirely on one field parameter, the mass coefficient k, with only a slight dependency on a second factor ϵ, the axial-loss factor. It is interesting to note that the theory of the actuator disk does not appear as a limiting case of the exact theory.

"Numerical Methods of Conformal Mapping," by G. Birkhoff, D. M. Young, and E. H. Zarantonello, contains a critical discussion of several methods, the most thorough treat-

ments referring to the three methods which the authors advocate most, for their simplicity and general applicability.

"Flow of Viscous Liquid Through Pipes and Channels," by J. L. Synge, deals with the numerical solution of certain boundary-value problems which occur in the theory of viscous flow. It might be described as a controlled method of finite differences, the control being supplied by the fact that at each stage of the approximation the solution is located on a known hypercircle in function space.

"The Method of Singularities in the Physical and in the Hodograph Plane," by Alexander Weinstein, deals with a generalized axially symmetric potential theory. It makes frequent use of an identity which permits obtaining from any stream function a potential function in a space of two higher dimensions. Results are given which are useful in making the results of electrostatics available for the method of sources and sinks in hydrodynamics, and elasticity, and vice versa.

ROBERT P. COLEMAN

HISTORICAL METROLOGY, by A. E. Berriman. 224 pages, illustrations, 13 × 20 cm. New York, E. P. Dutton & Co., Inc., 1953. Price, $3.75.

Where did the weights and measures we use today originate? Why do other peoples use different units and why do the same units differ from one country to another? These questions have been posed many times in the past. Now, as a result of a brilliant piece of archeological and historical research, the questions can be answered.

Historical Metrology is a book which represents a tremendous volume of research by someone dedicated to the uncovering of the origins of the weights and measures of the world. Mr. Berriman has, in a magnificent fashion, waded through original sources to find the correlations by which the present day measures are identified. He shows the interlocking of ancient measures and the evolution of present day standards.

The author states that this book arose from the casual observation that the royal cubit could be expressed by 20.6265 inches. As this number had geometrical significance in terms of the inch an investigation was begun which lasted ten years. In this period the author traced the origins of weights and measures from the ancient Babylonians down to the present day.

The investigations of the author indicate that ancient metrology was geodetic in origin. It appeared closely allied to the shape of the earth, for the circumference of the earth measures 216,000 Greek stades and the Greek stade measures 600 Greek feet. By a series of conversions there evolve from this information the geodetic mile and the knot. All through this work conversions of this type are demonstrated, which indicates the power and sweep of the author's analytical faculties.

One bit of historical significance will appeal. Only by the sheerest accident do we use the English system instead of the metric system for measurements. Actually, in 1790 there were discussions between England and France as to the advisibility of converting to a single system—the metric. In that year Talleyrand wrote to Sir John Riggs Miller suggesting the two nations adopt an equalization of measures. Unfortunately, the unrest in France came to a head in 1791 and by 1793 war broke out between the two nations. Had there been a stable government in France at that time the British Empire might have adopted the metric system and the United States would probably have followed.

For those who are interested in the origins of weights, measures and coinage this book is a must.

I. M. LEVITT

INORGANIC CHEMISTRY, by E. de Barry Barnett and C. L. Wilson. 512 pages, diagrams, 16 × 26 cm. New York, Longmans, Green and Co., Inc., 1953. Price, $7.00.

In this text, designed for either advanced or graduate students in chemistry, the authors have arranged the subject matter of the text on the basis of the atomic structure and occurence of the elements in the periodic table. The first six chapters of the book are devoted to a thoroughgoing discussion of the behavior of the elements as atoms; namely, natural and artificial atomic breakdown and its determination, isotopes and isobars of the elements, and the structure of the atom based on the Bohr-Rutherford concepts. Stereochemistry (the arrangement of atoms in tridimensional space) and crystallography are covered in one chapter. The isolation of the elements from their compounds is also discussed, but from the

standpoint of the chemical reactions involved rather than from the engineering processes. The remaining nineteen chapters of the book present the physical properties, crystal habits, and reactions of the elements, and the syntheses and properties of their compounds with the elements discussed in groups as they appear in the periodic table.

One of the most difficult tasks in chemical writing is to prepare a book that does not overlap heavily into the many branches of chemistry. The field of inorganic chemistry poses perhaps the biggest problem on three counts: (*a*) The close alliance of physical chemistry with inorganic compounds; (*b*) The many industrial and engineering processes involved in the isolation of the elements; and (*c*) The fact that the most important compounds (chemically) of carbon, hydrogen, oxygen, and nitrogen are the organic derivatives. Thus, the great problem facing the author of an inorganic chemistry book is to prevent his book from being a recount of physical chemistry concepts and data followed by a cataloguing of elements and both organic and inorganic compounds together with engineering details for their synthesis and isolation. While this book does not meet all of the aforementioned criteria for the perfect inorganic chemistry book, it does come closer to it than any other publication written in English. The authors achieve this by adhering strictly to a discussion of the elements on the basis of their atomic configuration, crystallography, and reactivity. Organic derivatives are discussed only as to how they enter into inorganic reactions. The authors are to be congratulated on preparing an advanced inorganic chemistry textbook that actually is concerned with mainly this field of chemistry. It is one of the few really advanced textbooks on this subject that is available. This book should prove to be a valuable addition to chemical literature in general, and it can be recommended specifically as a complete and modern textbook for advanced work in inorganic chemistry.

DONALD H. RUSSELL

INTRODUCTION TO SOLID STATE PHYSICS, by Charles Kittel. 396 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $7.00.

The aims and scope of this book are commendable, but one cannot help but feel somewhat disappointed in the product. There is at present a great need for introductory solid state texts. The major fault of this book is that of too much brevity. The author states in his preface that "the major problem . . . has been to conform to reasonable limitations on the length, in keeping with the spirit of an introductory textbook." From a pedagogical viewpoint this opinion is bad if the book suffers. Certainly this attitude is not found among those writing general physics texts and certainly they have the results of a great deal of experience in this field. There is no reason to believe that solid state physics is any easier for the senior or beginning graduate student than general physics is for the freshman or sophomore. It would seem that 343 pages is too few to a large part of the field of solid state physics. As a result of this space restriction, it appears as though the basic structure of fact and formula has been kept but that the explanatory meat and sinew have been stringently reduced. In many places the physical significance of phenomena has not been thoroughly discussed. The student is supposed to use his own immature physical insight.

There are many redeeming features in the book. It is really up-to-date and includes a great deal on modern techniques and many recent data. There are complete chapters on ferroelectricity, ferromagnetism and antiferromagnetism, superconductivity, semiconductors, and dislocations—fields in which there have been many recent advances. Each chapter begins with a brief description of what is to be covered and ends with general references and a number of excellent problems. Some of the problems are really an extension of the text. The text is well-illustrated and well-indexed. Throughout, there are specific references so that the interested student may easily further pursue particular topics. There are twenty-one appendices on proofs and advanced topics.

The book begins with a chapter on classification of solids and crystal structures. For a good understanding of crystal properties such a foundation is necessary. However, the author is presumably out of his element for there are a number of minor errors. The definition of atomic scattering factor is incorrect, the important words "coherent" and "Classical" have been omitted. The atomic scattering factor is not always Z at zero angle,

there is the possibility of the Hönl correction. The properties given for the fundamental translation vectors apply only to the primitive cell, a fact which is not stated. The term "lattice constant" is used but not defined. There is no clear differentiation of lattice and structure. The diagram of the rotation camera should have a small crystal at the camera center. The author used the Schoenflies crystallographic notation, although the Hermann-Mauguin notation has been adopted by the International Union of Crystallography. The unit cell is correct and elegant, but there is no accompanying homespun statement about its being a building block.

There are other faults throughout other chapters. ϵ is called the dielectric constant when most authors now call it permitivity. The depolarization factor is called a demagnetization factor and it looks as though the author intended to state that they were analogous. The short range order discussion never mentions other than nearest neighbor pairing. There have been many short range order parameters beyond nearest neighbors.

The space limitation is apparent in a number of places. There is an annoying number of references to figures elsewhere in the book. Values of Debye temperatures are given without any statement as to how they were obtained from elastic constants. A discussion restricted to longitudinal waves suddenly, without any explanatory transition, finds itself applied to transverse waves. At times the author forgets himself and thinks he is writing for a mature student. There are a few cases of awkward phraseology, such as "atoms not close in the periodic table to the inert gases."

In appraising this book one should judge it by its usefulness. With a good teacher to fill in, it should be extremely valuable. If the book comes out in a second edition, a fifty per cent increase in size would be welcome. We are disappointed that a man of Dr. Kittel's ability and stature in the field did not turn out a more polished work.

LEONARD MULDAWER

ELECTRIC CONTROL SYSTEMS, by Richard W. Jones. Third edition, 511 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $7.75.

The electric control of systems is too often neglected as a subject of study in electrical engineering curricula. If this has been so partially because of the lack of a suitable textbook, then this newly published work should fill the void. The author has tested the contents in the form of course notes, for a number of years, and has sufficient material for a one semester course, for senior students who have had courses in alternating and direct current circuits, differential equations, basic electronics and some machinery theory.

The sixteen chapters are organized functionally, there being a high degree of coordination between the principles of circuits, machinery, transients, etc. While the emphasis is given to controllers for electric motors, the principles are applicable to other power drive systems. Three chapters are devoted to the dynamics of motors and loads as integrated units of a system, their dynamic torque-inertia characteristics, and speed-time and speed-torque characteristics of a-c. and d-c. motors.

The next set of major elements in a controller system are switches. Five chapters consider: the operating characteristics of electromagnetic contact-making devices when applied as a part of a motor controller; gaseous and other electronic switching devices, including single and multiphase gas rectifier and inverter systems, power amplifiers, and trigger circuits, and switching circuit fundamentals. The latter subject is an introduction to the basic theory of switching networks and it should be one of the most interesting in the book for one who has not had the opportunity to read extensively on the subject.

In the remaining seven chapters, motor control circuits are synthesized with chapters devoted to: connecting and reversing; acceleration and starting characteristics; speed control by rheostat and switching methods; decelerating and braking; feedback control; and motor protection.

Throughout the text, illustrative examples are worked out analytically or graphically. The questions and problems at the end of each chapter should stimulate the reader's thinking as well as serve as drill and illustration for the text material. The extensive bibliography, while not exhaustive, is up-to-date.

The book may be used as a reference for engineers who have to design simple control circuits, in addition to its use as an undergraduate text.

S. CHARP

OUR NEIGHBOR WORLDS, by V. A. Firsoff. 336 pages, illustrations, 16 × 24 cm. New York, Philosophical Library, 1953. Price, $6.00.

Many attempts have been made to combine astronomy with space travel in a single book. *The Conquest of Space* and *Exploration of Space* are two of the better known examples which come to mind. However, neither of these go into the story quite as fully as Mr. Firsoff in *Our Neighbor Worlds*.

Mr. Firsoff is primarily an astronomer and so the book is heavily tilted in this direction with but 80 of the more than 300 pages devoted to space travel. In the astronomical discussion Mr. Firsoff deals primarily with the aspect of the solar system and the finished work is a tremendously interesting and detailed exposition of the system.

His survey of the solar system is in general in conformity with the accepted and most recent information available in the literature today. There are some deviations but only the professional will recognize these. The author seems to have neglected the astute work of Hertzberg on the atmospheres of the planets. Also, in the evolution of the solar system, the author describes the various theories which have been proposed but does not mention the work of Dr. Lyman Spitzer of Princeton who showed that in any collision or near passage of an intruder star the material pulled off, unless it had a mass of at least a tenth of that of the sun, would explode into space.

There are some places where the text could have been changed to lend the book authority. For instance, on page 92 the author, following Willy Ley, claims that ozone is spontaneously detonating. Recently this has been overcome by work at the Armour Research Foundation. Ozone is now considered stable.

There are many typographical and editorial errors. More than there should be if the book had been carefully proof-read.

On page 129 the author begins abbreviating miles per second and uses "ml" as the abbreviation of mile. It would have been better had he written this out. To readers on this side of the Atlantic "ml" stands for milli-liter. Dr. Wildt was formerly at Princeton but for at least 10 years has been at Yale.

On page 248 the body temperature is given at 89.6 degrees instead of 98.6. Chesley Bonestell is spelled correctly until the last chapter and suddenly there is an extra "n" tossed into his name. On page 129, in the table, we read 4.57 instead of 14.57 and there are other numerous transpositions of letters in words. This is an unfortunate carelessness, for this book is well written.

The author is a very clever writer with an amazing ability to present his story as a painting in words. While many books do the same thing, the brush usually used is a broad one which obscures the many interesting and fascinating details. Mr. Firsoff has painted his picture with a fine brush in which all the detail is disclosed and the story is a well rounded one.

This reviewer unhesitatingly recommends this book to all—the space travel enthusiasts, the young or amateur astronomer who would like to know about the details of the planets, and the general reader. I. M. LEVITT

BOOK NOTES

ALGEBRA: ITS BIG IDEAS AND BASIC SKILLS, BOOK II, by Daymond J. Aiken and Kenneth B. Henderson. 397 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1952.

This book is designed to serve as a text for a second course in algebra, definitely at the pre-college level. It stresses four "big ideas": numbers, logarithms, equations and functional relationships. Chapters on Trigonometric Functions and Progressions are included as well as some standard tables.

The presentation is from the teaching, not lecturing, point-of-view. The authors have departed from accepted tradition to address their readers informally. Illustrations border on the cartoon variety, and every effort is made to help the student learn. The authors tried this approach in Book I and feel its acceptance justifies using the same tactics in Book II. At the risk of insulting the bright ones, it may be just what most of the students need.

ALONG THE GREAT RIVERS, by Gordon Cooper. 159 pages, plates, 14 × 22 cm. New York, Philosophical Library, 1953. Price, $4.75.

With a gifted pen, Gordon Cooper brings to life ten of the world's greatest rivers in this small book. As a traveler, the author gives eye-witness descriptions of many of the great rivers and their basins to less fortunate stay-at-homes. In addition, he emphasizes the economic, social and historical importance of the rivers to the peoples who live along them. The whole story is told in kaleidoscopic fashion, with ever-changing pictures to interest the reader. The book is a fascinating mixture of history, geography, legend and colorful, factual descriptions. It is recommended as one of the better "travel" books.

PRACTICAL TAXIDERMY, by John W. Moyer. 126 pages, illustrations, 16 × 24 cm. New York, The Ronald Press Co., 1953. Price, $3.00.

Here is a useful guide for the amateur naturalist or hunter, who wants to mount his nature specimens. In addition to giving details on the preparations of mounts of birds, animals, fish and reptiles, the book contains suggestions for collecting the specimens, lists the needed equipment, and gives directions for preparing the formulas and solutions used. The author has written the book primarily for the beginner, yet the methods described will be of interest to professional taxidermists.

FIELDS AND WAVES IN MODERN RADIO, by Simon Ramo and John R. Whinnery. Second edition, 576 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $8.75.

In the nine years since the first edition of this widely used text, many new developments in the field have occurred. The authors have revised the original text to include: the mks system of units; additional problems; microwave networks; new material in the discussions of radiation, horns, slot antennas, receiving antennas and arrays. Also, some material has been deleted to make room for the new. The text is still aimed at the intermediate level. In its up-to-date form, the text will be welcomed by those who knew and used the first edition.

SIMPLIFIED DESIGN OF ROOF TRUSSES FOR ARCHITECTS AND BUILDERS, by Harry Parker. Second edition, 278 pages, diagrams, 13 × 20 cm. New York, John Wiley & Sons, Inc., 1953. Price, $4.00.

The second edition of this specialized design book has been prepared to incorporate the many recent changes in specification requirements relating to timber and steel. New tables are included and the examples have been changed to agree with current practice. An entirely new section on timber connections and their use has been added. Like the first edition, this revised one may be used for home study as well as in the classroom.

FUNDAMENTALS OF BIOLOGY, edited by M. J. Harbaugh and A. L. Goodrich. 611 pages, diagrams, 16 × 23 cm. New York, The Blakiston Company, Inc., 1953. Price, $6.00.

This new text has been prepared for a generalized college course in biology, and as such emphasizes general principles rather than details. Treating biology as the "science of life," the text covers living organisms in their simplest as well as in their most highly developed forms. Since Man is the most highly developed form of life, Man's place in the living world is treated in detail. The text should prove most useful for those students not majoring in biology, but who wish to understand the broad picture of the life processes in plants and animals.

50–100 BINOMIAL TABLES, by Harry G. Romig. 172 pages, tables, 19 × 22 cm. New York, John Wiley & Sons, Inc., 1953. Price, $4.00.

Included in this book are tables for the positive binomial $(q + p)^n$, where $q = 1 - p$. The tables cover the range of n values from 50 to 100 in steps of 5, and the range of p values from 0.01 to 0.99 in steps of 0.01. The tables were computed on a General Purpose Digital Computer at the Bell Telephone Laboratories, and should prove most useful in the field of quality control.

DATA FOR X-RAY ANALYSIS. VOLUME I. CHARTS FOR SOLUTION OF BRAGG'S EQUATION, edited by W. Parrish and B. W. Irwin. 100 pages, 21 × 30 cm. Eindhoven (Holland), Phillips' Technische Bibliotheek, 1953. (In English.) Price, $2.00.

DATA FOR X-RAY ANALYSIS. VOLUME II. TABLES FOR COMPUTING THE LATTICE CONSTANT OF CUBIC CRYSTALS, edited by W. Parrish, M. G. Ekstein and B. W. Irwin. 83 pages, 21 × 30 cm. Eindhoven (Holland), Philips' Technische Bibliotheek, 1953. (In English.) Price, $2.00.

These are the first two volumes of a series to be published under the heading "Data for X-ray Analysis" by Philips Laboratories, Inc. The charts of Vol. I give the value of d for any value of θ and of 2θ for Kα, α_1, and α_2 of Mo, Cu, Co, Fe, and Cr. θ is given directly to 0.01°; d is given directly to 0.01Å for values of θ below 15.5°, and to 0.001Å for values of θ above 15.5°. The tables of Vol. II give the values of $(h^2 + k^2 + l^2)^{\frac{1}{2}}$. $\lambda/2$ for values of $(h^2 + k^2 + l^2)$ up to 378 for Kα_1, α_2, and β_1, of Cu, Ni, Co, Fe, and Cr. The calculations seem to have been quite reliably made and the presentation of the data is clear. A few accompanying tables will aid in the use of the data. References to similar and associated material are given. The convenience, and hence the value, of these data is apparent to anyone engaged in X-ray crystallography.

Let's Stay in Business.—Six weeks after a $3,000,000 fire one day last winter totally destroyed it, a Philadelphia manufacturing plant was back in business—production resumed.

Term it a "modern miracle"—back in business in six weeks!

If it had been your plant, would it be in business today?

Would you be back at your old job?

The record shows there are some 70,000 such fires every year—not all, of course, so costly.

For instance, mercantile fires—those in the retail and wholesale trade, restaurants, bars, repair shops, filling stations and public garages—last year totalled 47,929. Those in manufacturing (factories, breweries, canneries, bakeries, laundries, printing and other manufacturing and processing) totalled 22,600.

There is no way of knowing just how many persons were laid off as a result of those fires. Nor is it recorded how many died in them—or as a result of them. Nor is it definitely known how many firms failed because of them.

Today, there is greater need than ever before for fire safety. As a result of the growing complexities of modern business, one industrial fire can seriously affect the livelihood of hundreds—even thousands—of persons. There are many ways by which this can happen; among them are these indirect but very real ones:

1. Loss of workers, who obtain employment elsewhere after fire results in plant shut-down.

2. Loss of customers—those who obtain their supplies from others because they cannot afford to wait for a razed plant to get back into production.

3. Loss of cash reserve resulting from costly repairs for fire damage.

4. Loss of competitive position brought about by curtailment of activity in research and development—activity which now must be spent on subsequent rebuilding and production changeovers.

5. Loss to the community through decrease in trade as a result of reduced earnings of workers living there; sometimes even increased tax burdens on you and your neighbors because property damaged by fire is removed from the tax rolls to the extent that it is not covered by insurance.

Certainly 70,000 fires should make us stop and think what we *individually* can do to stop them. For no firm can afford a fire and no one can afford to work for a firm that fails to provide adequate protection for its employees.

The National Board of Fire Underwriters contends that a practical program in fire safety, adapted to the needs of the particular business, will pay rich dividends, greatly reduce loss in life and property. Such a program, it points out, consists usually of five parts, as follows:

1. *Management Education.* Top executive to foreman must be firmly sold on the need for fire safety, must practice and enforce it.

2. *Inspections.* The essentials of the plant's overall fire protection must be reviewed periodically, must be kept from deteriorating.

3. *Fire Prevention.* This includes the elimination of fire hazards—it's good housekeeping.

4. *Fire Protection.* Pre-fire training of personnel for emergencies and other standard safeguards.

5. *Year-'Round Program.* Coordinated activities functioning day in and day out.

Electronic Control for Automatic Driving.—An exploration of how electronics can be put to work to reduce highway disasters and to relieve drivers of tiresome tasks on modern superhighways has been initiated by Dr. V. K. Zworykin, pioneer television and electronic scientist of the Radio Corporation of America.

Recent electronic advances, such as the tiny, power-thrifty transistor, indicate that electronic aids to many automobile driving problems are approaching the realm of practical application, according to Dr. Zworykin. Although the day of completely automatic control of automobiles is far off, Dr. Zworykin said, certain electronic devices to assist drivers in such matters as bad weather steering and collision prevention are nearer at hand.

To study the basic problems of automatic driving, Dr. Zworykin and assistants at the David Sarnoff Research Center of RCA, in Princeton, N. J., have equipped a model five-foot car with electronic equipment. This laboratory car, which is powered by a storage battery, can: 1. Steer itself along a prescribed route; 2. Stop itself when approaching a metal obstruction; and 3. Turn out of its original lane into a second lane as if to pass another car moving at a slower speed.

In the laboratory set-up, the model car is guided by a wire which represents a cable that would be laid in the roadbed of a superhighway. The wire sets up a magnetic field of a certain frequency which is picked up by the two coils, one on each side of the car. If one coil receives more of the signal than the other it means the car is no longer centered over the wire and electronic equipment controlling the steering wheel immediately brings the car back "on course."

To prevent a collision with an obstruction, simple transistor circuits associated with the guidance wire send out warning signals (of another frequency) whenever an obstruction passes or is stalled over them. These warning circuits, in effect, produce a "radio tail" at the rear of any sizable metal obstruction on the route. When equipment in the model car receives the warning signal, the brakes are automatically applied and the car comes to a halt.

To simulate two lanes in the same direction, Dr. Zworykin has parallel guidance wires with a diagonal wire connecting them. When the model car senses the radio tail of an obstruction in the inner lane, its electronic equipment shunts it along the diagonal into the outer lane so as to pass the obstruction.

A system of warning circuits in the roadbed to produce a "radio tail" when an automobile passes over would be quite impractical with electron tubes, Dr. Zworykin pointed out. Such circuits would be needed approximately every 20 feet and the electrical power to operate the tubes would be enormous, he said. But when transistors are available in large quantity at low cost, he said, such circuits become feasible because power consumption would be reduced a million fold over that required by tubes.

"The basic requirements of an automatic driving system harmonize with trends in modern highway construction," Dr. Zworykin continued. "The requirements are that the roads have at least two lanes in each direction and that crossings and left turns across traffic be eliminated by cloverleafs and similar systems. With these conditions satisfied, the stage is set for a gradual introduction of measures to reduce traffic risks and simplify driving procedures.

"The changes should necessitate neither sudden abandonment of estab-

lished driving habits nor wholesale installation of new equipment on roads and vehicles. This means: (1) The driver must retain the freedom of choice of speed, within prescribed limits, and of choice of either manual or automatic control. (2) Automatic control systems must be restricted, initially at least, to high-speed long-distance road systems subject to special regulation, such as turnpikes and thruways. (3) Vehicles equipped with automatic driving devices must be able to benefit in mixed traffic, consisting of equipped and unequipped vehicles.

"Freight transport along the highways presents one of the most challenging applications of full automatization. The establishment of a network of truck routes, separate from a highway system designed for passenger vehicles only, has been discussed repeatedly.

"With full automatization, it offers the possibility of driverless freight transport over long distances, with route terminals taking the form of marshalling yards. Here, small dispatching crews would send out the loaded trucks to their distant destinations and assign incoming vehicles to drivers for local delivery of the consignment. Assuming a cruising speed of 30 miles per hour, a separation between vehicles of 100 feet, and an average truck load of 5 tons, a one-lane route could handle nearly 200,000 tons a day. This large shipping capacity provided with minimum manpower requirements would do much to alleviate present bottlenecks in the delivery of consumers' goods and industrial raw materials."

New Hydraulic Pump Motor for Jet Bombers.—A new air-cooled 3½-hp. hydraulic pump motor is now being produced by the General Electric Company's Specialty Motor Sub-Department for use in jet bombers.

Rated at 7500 rpm., the motor supplies hydraulic power for control of the aircraft's tail assembly, and in the wing serves as a standby source of power where the jet engine is the primary source. It may be mounted either horizontally or vertically.

Of the ducted type, the motor employs an air-through design which utilizes a built-in blower to draw ambient air from outside the plane, circulate it through the brushes and windings, and expel it from the pulley-end stack. According to G–E engineers, the air-through design is much more effective than the old air-over arrangement because it allows air to be brought directly to the primary sources of heat, the armature and commutator.

The G–E motor can circulate air from minus 65 to plus 135° F. at altitudes approaching 50,000 ft. Explosion-resistant through the use of two blower tubes connecting via ducts to the outside air, the motor has a three-to-one gear reduction and is equipped with a radio noise filter. Condensate tubes on its side draw moisture out of the motor.

New Effect in Nuclear Reactions.—Research in the physics laboratories at the Massachusetts Institute of Technology has produced evidence that an actual collision of nuclear particles is not necessary for a nuclear reaction; the particles can react even though they merely brush past each other.

Dr. Clark Goodman, associate professor of physics, and Clyde McClelland, research assistant, believe that this is the first experimental evidence that nuclear projectiles need not score a bull's-eye to cause nuclear reactions.

Their results were announced recently in the *Physical Review;* the work was supported at M.I.T. by the

Office of Naval Research and the Navy Bureau of Ships.

"When a gasoline molecule bumps into an oxygen molecule and the two interact, chemical heat is produced which drives your car," explains Professor Goodman. "Similarly a nuclear reaction, such as those produced in a cyclotron, takes place when the bombarding particle strikes the core or nucleus of a target atom. The recent work at M.I.T. shows that nuclei can also undergo reactions without actually being struck."

In the M.I.T. studies, hydrogen nuclei—called protons—were accelerated by one to two million volts in a Van de Graaff-type electrostatic generator. These fast-moving protons, passing near—but not striking—the nuclei of any of several heavy elements, caused the heavy materials to give off high-energy X-rays.

The protons, says Professor Goodman, had such low energy that "they could never get near enough to do more than wave at the heavy metal nuclei"—yet they clearly produced high-energy X-rays (called gamma rays) which were measured and counted by a scintillation spectrometer.

"Putting this reaction on the same scale as our solar system," says Dr. Goodman, "it is as though a large meteorite, about half as big as the moon, whizzed by the earth at a distance of about 30,000 miles. The gravitational effect would create enormous wreckage on the earth's surface, even though our planet were never actually struck.

"In the same way," says Dr. Goodman, "the electrical effect of our fast-moving protons may be quite large though they never actually strike the nucleus."

These results, according to the M.I.T. workers, have been anticipated by the theoretical studies of several scientists, including Professor E. Guth at the University of Notre Dame and Professor Victor Weisskopf at M.I.T. Since the discovery at the Institute, the results have been confirmed by scientists in Denmark, at Duke University (Durham, North Carolina) and the Canadian atomic energy laboratories at Chalk River, Ontario.

Dr. Goodman emphasizes that the new technique offers no promise as a source of nuclear energy. But it does, he believes, "open the door a little wider to the secrets of nuclear structure which nature continues to hide so zealously."

These low-energy reactions are of particular interest in physical research because the amount of energy involved is so low that other nuclear effects are impossible. Thus there are no side-effects, and it is relatively easy to study the reactions.

The work reported, say the M.I.T. scientists, is typical of how new findings may come as fortunate—and unexpected—by-products of scientific activities. The original research was devoted to studying the reactions of one to two million volt protons with chromium and vanadium. The chromium and vanadium samples were mounted on plates of tantalum, and the first hint of the new effect came when unpredicted gamma rays appeared during the experiments.

It now turns out that tantalum was, under the conditions involved, the ideal metal to demonstrate the new effect. It has since been studied carefully in both tantalum and platinum, and preliminary work has been done with other materials including silver, tungsten, mercury, and bismuth.

Galvanized Shielded Enclosures.—A complete line of *galvanized* single-shielded enclosures for suppressing the

radio-frequency radiations of induction heating, dielectric sealing, diathermy, and other industrial and laboratory equipment, has been announced by the Ace Engineering & Machine Co., Philadelphia, Pa. The new Ace screened enclosures provide more than 40 db. attenuation from 14 kc. to 400 mc. Up to 66 db. attenuation at standard VHF television frequencies can be expected. This is more than ample for the large majority of industrial applications.

Ace Galvanized Single-Shielded Enclosures consist of a number of standard size panels rigidly bolted together from the inside. Each panel is a framework of kiln-dried, select white pine covered with a single layer of 22-mesh galvanized steel screening. The entire enclosure is easily erected, enlarged, or dismantled in just a few hours. Units are designed so that they may be readily installed to enclose a machine that otherwise would cause objectionable radio or TV interference. Standard floors are ¾-in. plywood with a loading capacity of 300 psi. Floors with greater loading capacity are available.

Virtually any kind of power, water, gas, or other service may be brought into the enclosure. Small access doors are available which speed production by allowing the continual transfer of material in or out of an enclosure.

Motor-Driven Transmission Line Shorting Switch.—A new high-frequency shorting switch for two-wire open transmission lines having 12-in. spacing has recently been introduced by the Trylon Tower Division, Wind Turbine Company, West Chester, Pa. Designed for flexible remote control in antenna "parks," the Trylon Shorting Switch can be used for short circuiting two or more frequency tuning stubs; shorting transmission line accept-reject circuits; or as a grounding device for transmission lines or other high voltage a–c. or d–c. circuits. The switch causes a minimum of line disturbance in the "open" position and is conservatively rated to handle 30 KV. wire-to-wire or 15 KV. to ground.

The heavy steatite-insulated shorting bar is controlled by a low-voltage d–c. motor equipped with limit switches and an electro-mechanical brake that stops the motor instantly at the end of the stroke. The mechanical link between the motor and the switching arm disengages automatically at the end of the stroke so that the switch may be hand operated in case of power failure. The motor is completely weather-and fungus-proofed and equipped with built-in gear reduction. Control consoles incorporating a signalling system to indicate switch operations can be designed to suit individual requirements.

New Blasting Technique.—Igniter cord, a new device safer and more efficient than older methods, has been adopted by The International Nickel Company of Canada, Limited, for use in drift driving at its mines in the Sudbury District of Ontario. In use now after exhaustive tests at Inco's mines, igniter cord requires a blaster to remain at the loaded face only long enough to light a single fuse, instead of having to light the fuse in each drill hole separately. Igniter cord has been found to be more satisfactory than electricity for detonating the holes.

The chief aim in International Nickel's development of igniter cord was the safety advantage gained through having to light only one fuse. This is particularly desirable where the path of retreat from the working place is difficult and time-consuming, or where staging has to be removed.

With the use of igniter cord it is possible to delay the actual lighting of the round for a considerable time by employing a fuse of sufficient length as a "master fuse" so that the igniter cord circuit is not lit until everyone has reached a place of safety.

All fuses in a loaded face are connected to the igniter cord in the order in which it is desired to detonate the holes. When lit, the igniter cord burns along its length with an external flame at the zone of burning. The flame, usually short and white hot, in turn lights the safety fuses in rotation. The type of cord used at the Inco mines burns at approximately 20 seconds per foot, and is designed to meet the conditions encountered in development rounds or locations where the holes are drilled relatively close together. Another type of cord with a burning speed of about eight to 10 seconds per foot is designed for use in conjunction with safety fuse in achieving rotation firing on stope breasts where the holes may be collared from two to four feet apart.

The safety fuse is attached to the igniter cord by means of a connector in the form of a shell with a composite ignition charge consolidated under pressure. A slot at one end of the connector holds the igniter cord securely.

New Orthochromatic Sheet Film.—A new, fast orthochromatic sheet film was announced by the Du Pont Company at the opening of the annual Photographer's Association of America convention in Chicago.

Designated "High Speed Ortho," the film meets the particular needs of portrait and press photographers who require a fast film, combined with strong tonal rendition offered by orthochromatic sensitivity. Ortho film is especially suitable for portraits of men and children.

Studio photographers will also find that the new film's overcoating makes retouching possible without additional preparation of the negative. The high speed emulsion, of course, permits short exposures.

For press-type pictures taken under adverse exposure conditions, the film's fast speed combined with low red sensitivity provides good contrast, with the inspection method of development allowing some compensation for poor lighting conditions. A dim red safelight may be used for inspection.

High Speed Ortho gives good separation of tones on blues and greens, a desirable characteristic for most landscape photographs, as well as for many aerial views.

Having high sensitivity to blue, virtually none to red, and less to green than panchromatic film, High Speed Ortho produces excellent results on tungsten-lighted banquet scenes.

With medium speed "X–F" Ortho, the new film now makes available a selection of high quality Du Pont orthochromatic emulsions suitable for a wide variety of lighting and exposure conditions.

Photographic Method of Graduating Precision Instrument Dials.—Generally there are three prime sources of error in divided circles or dials, namely non-uniformity of spacing, non-uniformity of line width, and eccentricity between center of generation and center of rotation. To produce dials of the highest precision, the errors from each of these sources must be rigorously brought to an absolute minimum.

Precision photomechanical engineers have arrived at a scientific departure from any known method of producing precise markings by means of an optical-mechanical recording in-

strument. There is no possibility of error in uniformity of spacing or character, because this method automatically generates optically from a single-line master pattern.

Dial graduations can be made of any width from as fine as three microns up to any size desired to meet commercial requirements. The master dial is generated on a glass plate that is ground optically flat before it is made light sensitive. Its dimensional stability therefore becomes that of glass. Photographically produced replicas are made by contact printing under vacuum and dial blanks printed from the master, using a center hole of the same tolerance as the hole in the generated master itself, insures dimensional stability and sustained accuracy that cannot be achieved by any other method of production. These dials may be of any size, color or combination of colors and any style or type of legend may be permanently fashioned as part of the dial itself. Also they may be produced on any metal or plastic material ready for installation.

Color Film on Nuclear Fission Available.—The new movie "A Is For Atom," hailed as a masterpiece of visual education, is now available to business and industrial concerns, the General Electric Company has announced.

Reviewed and endorsed by the Atomic Energy Commission, the color film explains the phenomenon of nuclear fission, and highlights the multitude of potential benefits in medicine, industry, agriculture and other fields.

A 10-minute version of "A Is For Atom" being shown in key movie houses throughout the country has received high praise from critics. The longer 16-minute version of the film, which is now offered to business and industry, goes into greater detail on the subject of atomic energy.

The film will be widely used in schools and colleges as a summary of present knowledge of the application of atomic energy.

Business and industrial concerns interested in using the film for employee or public showings may obtain it on loan from film libraries of G–E district offices, or may purchase a copy at the print cost of $100.

Photography in Body Cavities.—Speculum Speedlight was invented to take guesswork and painstaking focussing out of closeup photography for physicians, surgeons and dentists working in body cavities. It gives accurate color reproduction of ear, rectum, vagina and oral conditions with previously unattainable ease and in a matter of seconds.

Made by Medical, Dental, Scientific Photographic Equipment Co. (Lynn, Mass.), Speculum Speedlight does away with old-fashioned lighting setups. Instead of cumbersome floods it uses a one-inch ring light of great intensity and of adequate brilliance for perfect color work. The flash duration, 1/1000 of a second, freezes action so that there is no blur. Quick or rapid breathing and rapid pulsing or tissue movement are arrested by the flash in sharp image.

This clinically-tested unit was specifically designed for the doctor, the dentist and the scientist. Focus is fixed for any given field. Another great advantage over any previous close-up device is the incredibly light weight of Speculum Speedlight.

The instrument is a speedlight—frequently, though not accurately called a "strobe," or stroboscopic, flash. The one-inch circular flashlamp, or ringlight, which illuminates the field to be pictured, fits around the

outer rim of the camera lens. This is coupled, by means of an adaptor ring, to the viewing end of a speculum, so that it lies between speculum and camera.

With the field centered, the doctor, surgeon or dentist need merely touch the camera release button. This both trips the camera shutter and touches off, or activates, the brilliant flash ring. These are perfectly synchronized. The light perfectly illuminates the chosen field, through the speculum—with shadowless, perfectly-balanced intensity bound to produce flowless color slides in the finest detail. It employs "cold light"; generates no heat, does not subject a patient to discomfort.

Speculum Speedlight can be used with either a tripod or can be operated easily from the hand. The Rings add only about three ounces to camera weight, a tremendous reduction compared with any other photo-lighting set-up for critical close-up work in body cavities.

Speculum Speedlight is available to fit most speculums now used in proctology, in gynecology or in ear, nose and throat. Fixed focus is obtained by incorporation in Speculum Speedlight of a special close-up proxar lens which eliminates the need for focussing. Because the ring-light affords such even, brilliant intensity for color work, the camera's taking lens can be shut way down to provide utmost depth of field, another factor to ensure perfect focus. Guesswork is eliminated.

Scientific Translations Center.—A center for holding and photoduplicating foreign scientific translations has been established by the National Science Foundation in the Science Division of the Library of Congress. Partial support for the project has been contributed by the U. S. Atomic Energy Commission. Establishment of the translations center is part of a Foundation program to assist American scientists in keeping informed about current scientific progress in other countries.

Scientific translations will be collected from many sources, including Government agencies, scientific societies, industrial laboratories, and universities. Initially, the center will put major emphasis upon translations from Russian scientific journals, although it is hoped eventually to add material from other languages for more comprehensive coverage of the world's scientific literature. Monthly lists of translations newly received will be issued by the Library of Congress. The monthly listings will include notices of translations that are available by direct purchase from commercial and other translating services, but the center will not supply photocopies of such material.

Spectrograph for Modest Budgets.—Bausch & Lomb Optical Co., of Rochester, N. Y., announces a versatile new 1.5 meter stigmatic grating spectrograph designed to bring precision spectrography within the reach of small firms, hospitals, colleges, and even high schools. The instrument is available in two models which provide different dispersions, resolving powers, and plate coverages. Both are capable of analyzing a wide range of nonferrous materials, and, in addition, are suitable for use on the more complex spectra of unalloyed gray irons, plain carbon steels, and low-grade ores.

In tests during its development, the instrument was found to produce a spectral line quality equal to that of far more elaborate equipment in analyses of brasses, aluminum, magnesium, and bearing alloys, zinc base castings, lubricating oils, ceramics, pigments,

dyestuffs, biological materials, pharmaceuticals, and inorganic chemicals. Tests on an iron arc spectrum showed, under high magnification, clearly defined resolution of the fourth component in the 3100 group. This resolution of the "iron triplet" is regarded by spectrographers as a critical test in determining instrument quality.

Weighing only 150 pounds, the instrument is 12 inches high, 18 inches wide, and 60 inches long. The spectrograph's compact size and freedom from moving parts and adjustments were designed to meet the problems of small firms and institutions and the needs of inexperienced spectrographers.

The instrument will also serve a valuable function as an "extra" spectrograph in the laboratory. It can handle efficiently one-time analyses and eliminate the necessity for breaking down set-ups on larger instruments.

In both new models, the optical system consists of a concave 40 × 80 mm. Bausch & Lomb Certified-Precision grating, slit, quartz cylindrical lens, and film. The combination of cylindrical lens and concave grating provides a stigmatic spectrogram and permits the use of accessory equipment at the slit.

New Attachment for Welder's Mask Patented.—A patent grant for an attachment to a conventional welder's mask for cold weather operation was awarded recently to Mr. Curtis L. Olson, Deputy Chief of the Mechanical Equipment Branch at the Army's Engineer Research and Development Laboratories, Fort Belvoir, Virginia.

Mr. Olson's development, easily attached to a conventional mask or hood, is designed to prevent the lens on the mask from frosting in freezing temperatures. Present masks frost in cold weather and make operation of welding equipment difficult.

The new attachment can also be applied universally. Tests have indicated that it also improves operator comfort in hot weather since its ventilating effect provides for the removal of stale air and welding fumes from inside the helmet.

Remote Handling Tongs.—The Atomic Center for Instruments & Equipment, Inc., New York, has introduced the Atomlab Radiarm, Jr. Remote Handling Tongs to be used by the radiochemist, radiologist, and health-physicist for handling radioactive materials.

The Radiarm, Jr. pistol handle gives good leverage and a comfortable grip when handling heavy or light objects which are held by serrated jaws under firm spring tension. Accidental dropping is prevented by positive spring pressure requiring a deliberate finger squeeze on the trigger to release the object. The jaws can be locked into a fixed position by tightening, with thumb action, the knurled nut at the pistol grip end.

The Radiarm, Jr. has an over-all length of 19 in., weighs 1 lb. and is well balanced. The tongs are made of aluminum, which lends itself to easy decontamination.